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CONTENTS

Films and Television 393
Their Use in Mathematics Education . . . . . F. A. Ficken 393
Their Status and Future in Mathematics Education . P. S. Jones 403
Spies, Electric Chairs, and Housewives . . . P. L. Chessin 416
At the University of Alabama . . Ayrlene McGahey Jones 421
At the University of the Andes . . . . . A. Zawrotsky 424
At the University of Houston . . . . . Martin Wright 425
At the University of Maryland . . . . . R. A. Good 426
At State Teachers College, Montclair . . . . . B. E. Meserve and E. M. Maletsky 427
At Northeastern University . . . . . Joseph Spear 429
At Purdue University . John Dyer-Bennet, W. R. Fuller, W. F. Seibert, and M. E. Shanks 430
At Washington University . . . . H. Margaret Elliott 440
At the University of Washington . . . C. B. Allendoerfer 444
Elementary Problems and Solutions 446
Advanced Problems and Solutions 451
Recent Publications 456
News and Notices 468
The Mathematical Association of America 471
New Members. 471
New Department 473
November Meeting of the Philadelphia Section 474
February Meeting of the Louisiana-Mississippi Section 474
March Meeting of the Southeastern Section 476
March Meeting of the Southern California Section 482
Calendar of Future Meetings 484

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The AMERICAN MATHEMATICAL MONTHLY

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THE USE OF FILMS AND TELEVISION IN MATHEMATICS EDUCATION*

F. A. FICKEN, University of Tennessee

ORGANIZATION AND ACTIVITY OF THE BOARD

Educational institutions are facing a tremendous challenge in maintaining high standards and quality of instruction while accommodating vastly increased enrollments. Able and inspiring teachers are urgently needed at all levels of instruction. Far greater use will be made of various technological aids to education.

Films and television are already widely used for educational purposes and their use is increasing at an accelerated pace. The number of operating educational television stations, for example, has increased from one in 1950 and three in 1953 to 27 in 1957.

The Advisory Board on Education (ABE) of the National Academy of Sciences—National Research Council (NAS-NRC) took cognizance of this activity, and decided as a first step to study these media as they may be used in mathematics education. A Film (and Television) Evaluation Board (FEB), appointed through the Division of Mathematics of the NAS-NRC, was charged with the responsibility of developing criteria on content and objectives for production and for use of filmed or televised mathematical lectures. The FEB was also requested to recommend a course of action appropriate for the ABE as part of a continuing program within the NAS-NRC, relating to the use of such media in education in mathematics, and in other areas of science.

The FEB met at the Pennsylvania State University on August 21–24, 1957. The appointed members consisted of F. A. Ficken (University of Tennessee), chairman, and A. M. Gleason (Harvard University), T. H. Hildebrandt (University of Michigan), G. Hochschild (Institute for Advanced Study, Princeton, and the University of Illinois), J. D. Mancill (University of Alabama), and B. E. Meserve (State Teachers College, Upper Montclair, New Jersey). The meeting was also attended by P. S. Jones (University of Michigan), who is Chairman of the Committee on Educational Films of the Mathematical Association of America, by P. A. Smith (Columbia University), who was then Chairman of the Mathematics Division of the NAS-NRC, and by R. M. Whaley, who is Executive Director of the ABE of the NAS-NRC. The FEB also enjoyed the helpful counsel of C. R. Carpenter and A. W. VanderMeer, who are well acquainted with the extensive activity in film research and production at the Pennsylvania State University.

* This paper consists of a précis of the report of the Film Evaluation Board (FEB) of the Advisory Board on Education (ABE) and the Division of Mathematics of the National Academy of Sciences-National Research Council (NAS-NRC). The complete report is Publication 567 of the NAS-NRC [Library of Congress Card No. 58-60009], and may be purchased for $1.00 from the Publications Office, NAS-NRC, 2101 Constitution Avenue, Washington 25, D. C.

The Board viewed over thirty-five recorded films, selected for variety of content, mathematical level (rudimentary arithmetic to college algebra and trigonometry), intended audience, and general objectives (motivation, auxiliary to a course, or part of a course).

The work of the FEB was supported by a grant from the Fund for the Advancement of Education.

The report of the FEB has been accepted by the ABE of the NAS-NRC, and action is being taken to carry out the recommendations.

GENERAL OBSERVATIONS

By a "presentation" we shall mean a single showing of a film (in one place or over television) or of a television program (live or recorded, closed-circuit or broadcast).

A presentation in which an outstanding teacher achieves his best performance can offer a receptive student more skillful exposition than a "normal" classroom lecture; teachers, moreover, can learn from such a presentation some ways in which to improve their own work.

The new media make it possible to offer academic instruction beyond the technical capacity of conventional means.

The new media, especially television, provide an opportunity to offer public presentations in which the nature of mathematics and mathematical activity can be clarified and attractively illustrated; enlightenment can be made entertaining in the best sense of the word.

The new media make it possible, finally, for the viewer to witness objects, scenes, and occurrences at distant times and places. Motion and change in visible and audible data can be displayed in ways not otherwise possible. Phenomena invisible or inaudible to unaided senses can be brought above the thresholds. The possibility of producing a permanent record justifies elaborately contrived demonstrations, perhaps otherwise prohibitively expensive or destructive.

One unfavorable possibility, on the other hand, lies in the wide dissemination of erroneous ideas and unfortunate pedagogical stereotypes. Mass media entail a heavy responsibility; a single misunderstanding communicated in a presentation to a large group of students can handicap the efforts of all the teachers who must deal with the students personally. Large investment in equipment and recordings moreover, may strengthen resistance to expensive improvements.

Serious problems are raised by recorded sequences, in which a very substantial share, at least, of the expository burden of an academic course is allocated to presentations. It will not be easy to arrive at a favorable division of responsibility between presentations and various more personal methods of instruction. The new media should be used to supplement and improve instruction rather than to displace teachers. Another problem centers on proprietary interests; a recorded sequence is a substantial product.

RECOMMENDATIONS

A. Recommended Action.

1. *Creation of a Standing Committee on Mathematical Films and Television.* The Film Evaluation Board recommends the establishment, within the Division of Mathematics of the National Academy of Sciences—National Research Council but fully connected with and perhaps reporting to the Advisory Board on Education, of an adequately-staffed Standing Committee on Mathematical Films and Television. It appears to the Board that the functions of the proposed Standing Committee should include these:

a. Collecting, classifying, storing, and disseminating reliable data on existing mathematical presentations (both domestic and foreign, at all levels), including trustworthy indications of content, duration, purpose, appropriate audiences, and likely value for each purpose to each audience, in such a way as to facilitate advantageous choice and use of these presentations.
b. Promoting well-designed research on means of evaluating mathematical presentations and of principles for guidance in their effective preparation and use, research on the technical problems of feedback, and on other emerging problems.
c. Promoting the active participation of professional mathematicians in planning, refereeing, presenting, and reviewing presentations having significant mathematical content or significantly related to our profession.
d. Assisting in the development of equitable and realistic economic policies with reference to the proprietary interests of those engaged at any stage in presentations, and with reference to the intellectual and economic impact of films and television on teaching and on teachers.
e. Providing liaison with the mathematical community, with related professional groups, and with interested commercial enterprises; in particular, collecting and distributing information on emerging needs for presentations and on advantageous uses of them.
f. Publicizing its own existence and activities.

The Committee should avoid any action which would give it the status of a certifying agency.

The Film Evaluation Board estimates that such a Standing Committee* can be of workable size and still reflect regional differences within the nation, and a wholesome variety of professional experience and attitudes, as well as the views and interests of the American Mathematical Society, Mathematical Association of America, National Academy of Sciences, American Association for the Ad-

* Following acceptance by the ABE of the FEB report, the Division of Mathematics of the NAS-NRC has appointed the following standing Committee on Mathematical Films and Television: C. B. Allendoerfer (University of Washington), Chairman, R. A. Good (University of Maryland), G. A. Hedlund (Yale University), P. S. Jones (University of Michigan), J. R. Mayor (American Association for the Advancement of Science), E. J. McShane (University of Virginia), P. C. Rosenbloom (University of Minnesota).

vancement of Science, National Council of Teachers of Mathematics, and other organizations having a basic interest in mathematical education.

2. *Action for Other Areas of Science.* The Film Evaluation Board recommends that the National Academy of Sciences—National Research Council consider steps to ensure that the functions listed above be performed also in other areas of science, perhaps wholly or partially by similar standing committees.

B. Recommended Policies.

1. *Qualitative Considerations in Planning and Producing Presentations.* The presentations viewed by the Film Evaluation Board embodied various mathematical and pedagogical values which were a prime concern of the Board. Our views are set forth as supporting argument. The FEB recommends that these and similar considerations be kept in mind in planning and producing mathematical presentations.

2. *Proprietary Interests in Presentations.* The Film Evaluation Board recommends that whenever a presentation is exhibited an equitable dividend should accrue, as a matter of right, to all persons who participated in any phase of the presentation. Participation in some phase of a live educational presentation on television by a member of the staff of an educational institution, with no thought of later presentation, should be treated as an appropriate fraction of his academic duties; if a kinescope is made, an understanding should be reached, prior to his participation, regarding his interest in the event of later repeated presentation. The foregoing proprietary interests, as well as those relating to supplementary materials, professional acknowledgments, and so on, should be embodied in a negotiated contract before work is begun.

3. *Policies Regarding the Use of Presentations.* The Film Evaluation Board recommends that presentations be used in such a way as both (a) to promote frequent direct association between students and active scholars and (b) to promote healthy growth of younger teachers toward mature ability and professional dignity. With reference to an extended series of presentations offered in connection with a more or less conventional academic course, it is the unanimous opinion of the Board, pending further experience, that not more than half the time allotted to formal group instruction should be used for presentations, that such presentations should be devoted primarily to the exposition of basic ideas and principles (with a few well-chosen illustrations but no repetitive manipulation), and that the rest of the time should be used in accordance with the recommendation in the preceding sentence.

SUPPORTING ARGUMENT

A. Recommended Action (omitted from précis).

B. Qualitative Considerations in Planning and Producing Presentations.

Many principles of effective teaching, and even the very spirit of mathe-

matics, were violated in some of the presentations viewed by the Film Evaluation Board and we can offer only a brief summary of our observations. We emphasize at the same time our opinion that the very substantial outlay required to produce a program dictates extremely attentive preparations, and the further opinion that costly technical features often add less educational value to a program than the comparatively inexpensive services of able collaborators.

Our statement follows the chronological order in which a presentation is produced and used. With certain *data* in mind, the production is *planned*. After suitable *preparations*, a *live performance* occurs before film or television equipment. Whether or not a recording is made, certain *further steps* may be desirable.

1. *Data*. Certain data must be fixed, by choice or circumstance, before intelligent planning is possible. The presentation has an aim, an audience, and certain technical resources for use in its production. The primary aim of an educational presentation is to arouse and nourish thoughtful interest and activity. The intended audience may vary widely in vocabulary, information, experience, interest, susceptibility, motive, reaction-time, duration of attentiveness, and so on. The technical resources include the capabilities of the equipment and its operators. Other technical variables which may be fixed before planning begins include auxiliary audio-visual equipment and such parameters as the duration of the program, time and context of intended presentation, and so on.

2. *Planning*. Planning is essentially a collection of choices among varieties of substances and treatment, of emphasis, and of talent. These choices should take full account of the data. Language and substance should be correct and accessible to the audience, whose preparation should permit it to respond to most of the amount offered at the pace at which it is presented. The level during most of a program should be low enough for the estimated "average" of the audience, but high enough in occasional clearly labeled digressions to interest more sophisticated viewers as well.

Certain principles regarding substance and treatment apply to all presentations. Correctness must be impeccable. Mere freedom from obvious blunder is not enough; the reflective viewer who tries to push his thinking further must not be led astray by erroneous implications. The material must give a sound general impression, in harmony with the most enlightened professional understanding. The treatment should bring to light the fundamental intrinsic significance of the facts and relationships presented.

Any presentation should attempt to communicate basic principles and ideas, along with genuinely illuminating illustrations. The capacities of the media to portray motion and change in visible and audible data should be exploited fully. The Board was particularly impressed with animated drawings, and feels that their wise exploitation would yield valuable educational dividends.

Public presentations should emphasize motivation and entertainment, while maintaining significant contact with genuinely mathematical ideas. The aim should be to convey information of cultural interest and conceptual relationships

rather than technical skills; extensive manipulations should be avoided.

Presentations auxiliary to an academic program should have substance and treatment appropriately related to the material they supplement, serving objectives somewhat similar to those of visiting lecturers, and should be used only if they convey the intended idea in a superior way. Routine manipulations should be kept to a minimum.

An extended series of presentations, bearing a significant part of the primary expository burden of an academic course ("recorded sequence"), should improve on standard presentations, amplifying and elucidating the text. While some manipulations will be unavoidable, no attempt should be made to replace practice by the student. New courses and curricula present special opportunities for the experimental development of original discussions which may be very suitable, after modifications indicated by experience, for presentation on film or television.

The primary aim of a presentation is educational, to stimulate active mathematical thought, or at least a discriminating interest in it. The emphasis will therefore be on challenging the viewer, on stimulating his curiosity. All presentations should appeal to active reason, thus enhancing the viewer's confidence in his own rational processes. A supreme effort should be made to convey the intrinsic interest and value of the subject, and the pleasant excitement of seeking and gaining understanding for its own sake.

The speaker should display genuine authority with a lively but not exhausting personality. Especially, perhaps, in a public presentation, he should be identified as an active scientist with a recognized gift for communication. A speaker of unusual ability should ordinarily be engaged even for elementary material.

Unlike commercial presentations, whose principal goal is acceptability to the audience, an educational program strives for acceptability only secondarily, in order to advance the quite different primary aims indicated above. It lies in the nature of organized education that the knowledge and appreciations of the teacher shall surpass those of the viewer, whether student or layman. The value of an educational program is therefore determined only in part by its acceptability to the viewer; to be of educational value it will ordinarily have features to which his standards of acceptability do not apply, but which are subject to the considered judgment of experts. For this reason the aid of an expert consultant or referee is indispensable both in planning and in the ensuing preparations. He can detect glaring defects in substance and treatment not noticeable to most viewers, nor even to experts in the media. He should be rewarded accordingly; like the expert speaker, an able referee will cost little compared with the total outlay. Presentations making significant contact with other areas (*e.g.*, physics, engineering, social studies) should be monitored also by specialists in those areas.

3. *Preparations.* Motivations must be honest. There are many problems,

even some with quite practical aspects, having genuine intellectual appeal, capable of conveying to many viewers in an exciting way the power and elegance of our subject.

Illustrations should be clean. An example may leave a very foggy impression if it involves many features whose relation to the ideas at hand is not clear, thus leaving the viewer in some doubt as to just which features are illustrating which ideas.

Response to viewer's reaction must be judicious. The reactions of a "guinea pig" representing the "average" viewer may supplement helpfully those of the expert referee.

A very great amount of time may be required for thoughtful adjustment of content, style, and displays to the particular aim and audience intended.

Auxiliary preparations include suitable written materials, audio-visual aids, forewords to class and collaborating teachers, and so on. Technical preparations must ensure that sight and sound and esthetic appeal will be above the thresholds of acceptability.

In both preparation and live performance it is important to observe the difference between entertainment and distraction. Both films and television have been accepted by most viewers, before witnessing an educational presentation, as pastimes yielding in many instances mere distraction or, at best, entertainment of a somewhat frivolous kind. It is thought that sensitive judgment can maintain an entertaining atmosphere in an educational presentation while avoiding features which will divert the viewer's attention from the fundamental aims of the program.

4. *The Live Performance.* Visual displays must be visible, adequately lighted, and not obscured by perspective or by the speaker's shadow. The camera should not dwell at tedious length on the speaker.

The speaker should be lively and enthusiastic but relaxed in voice and manner, should avoid both apologetic and patronizing manner and language, and should expect and merit the respectful attention of the viewer.

5. *Further Steps.* The need for research on methods of evaluation indicates that thoughtfully prepared follow-up studies of individual presentations will be essential.

An instructional presentation will presuppose certain well-defined responsibilities of students and of collaborating teachers. It is essential, in particular, that a student have ample means, after a presentation, for determining (perhaps for himself) what he has understood and can deal with confidently, and what he has not understood. He must have ready access, in time of need, to clarifying personal guidance.

C. Proprietary Interests.

The novelty of presentations, especially of television programs and recordings of them, implies tentative and negotiable economic arrangements. One

factor determining a collaborator's reward is clearly the extent of his responsibility; is he, for example, composer or performer, or both? Economic forces, left to themselves, would ultimately produce decisions on many of these questions; indeed, they have yielded mutually-acceptable relationships between, for example, authors and publishers. We believe that the proposed Standing Committee should take cognizance of the evolution and stabilization of the economic rights of collaborators in presentations, in order that equitable and realistic policies may be reached as quickly and easily as possible.

D. Intellectual and Economic Impact of Recorded Sequences on the Teaching Profession.

The FEB holds firmly to the belief, which admittedly still lies beyond objective demonstration, that a student's growth toward professional stature is enormously stimulated by personal contact with scholars of professional ability engaged in professional activity. We are also concerned to secure the healthy development of further generations of teachers and scholars, observing that our society now offers permanent support, with minor exceptions, only to the *teaching* activity of a pure scholar in any field of learning.

We may distinguish between personal and group instruction according as the teacher can or cannot adjust his instruction, deliberately and immediately, to the specific personal characteristics of individual students. Presentations are essentially means of indirect group instruction.

Carefully prepared public presentations and occasional presentations supplementing and enriching conventional direct group instruction can clearly have great value. These uses of presentations are comparatively well-understood, at least in principle, although the need persists for improvement in the production and use of presentations. Extensive systematic use of presentations as a primary means of group instruction, while in a sense possible before the advent of television, has since then been enormously stimulated.

We shall speak of a recorded sequence whenever a substantial part (not necessarily all) of the mission of group instruction is allocated to an extended series of presentations produced for the purpose. We are thinking primarily of indirect group instruction in high school and at the lower-division collegiate level.

A fully recorded course makes feasible indirect group instruction not possible by conventional direct means; for example, television can reach sparsely distributed students, the bedridden, the snowbound, and so on.

A recorded sequence makes it possible to entrust one teacher (the speaker in the recordings) with the corresponding part of the expository responsibilities toward a large group of students normally borne by each teacher toward his own students. It is then possible to meet a given commitment for group instruction with fewer live teacher-hours in the classroom. Example: 12 sections, 3 hours weekly, require 36 teacher-hours in the classroom; if all sections view a presentation simultaneously during one of the three hours then, quite crudely,

only 25 teacher-hours in the "classroom" are required.

There is little doubt that the more specific, more tangible needs of group instruction can be met acceptably by recorded sequences of sufficiently high quality. This generally favorable estimate raises in some minds an appealing fantasy of "best" teaching, in which the unique definitive exposition is installed on film and tape and distributed widely, with a revised version appearing once every decade or so.

It must be remembered, however, that even the most successful expositor fails to reach some entirely worthy students, and in any case will not arouse uniformly the enthusiasm felt by those with whom he is most successful.

Pending the development of really effective and convenient feed-back devices, moreover, a live lecture has an undeniable advantage over an equally good presentation, in that interruption and explicit interchange between class and teacher is in fact possible, whether or not this facility is used advantageously being another question.

Even without interruption, finally, an experienced lecturer who can see and hear his audience, even a large one, can often sense their attitude and can adjust to it, stimulating them, so to speak, at a resonant frequency.

Insofar as presentations may be less responsive than direct group instruction to the needs of an individual student, and insofar as presentations may achieve less fully the tangible and intangible goals of group instruction, to that extent each student may need more and better personal instruction than he would need if all group instruction were direct.

Personal contact between students and teachers is important at all levels of instruction. Such contact is already regrettably scanty in many situations and threatens to deteriorate further as mounting enrollments outstrip the supply of qualified teachers. Recorded sequences should be used in such a way as to promote personal association between students and teachers; some of the eleven "released" hours in the example above should be devoted to strengthening the program of personal instruction.

Recorded sequences may not only increase the need for personal instruction, but may also change its nature and the qualifications of those entrusted with the responsibility for it. The personal teacher must understand the presentation fully (and may therefore need to know more) and may have to respond more sensitively than now to differences between individual students. Perhaps special training will be needed in giving personal instruction accompanying recorded sequences.

A recorded sequence may decrease the amount of group instruction required at a given time in a given institution. Unless the program of personal instruction is correspondingly expanded, as would indeed seem most desirable, a temptation may be felt to continue employing prospective teachers, in order to help meet their economic problems, but to use them primarily for menialities such as marking papers, taking attendance, calculating grades, and so on. Such a policy

would deny prospective teachers a rewarding share of the actual conduct of direct group instruction.

The Film Evaluation Board is firmly convinced of the present and future value to students and to the community of an alert, well-informed, proud corps of teachers. Scholars and teachers nourish and transmit our culture. Their economic rewards are comparatively small. Further decline in their intellectual or moral position would be disastrous at the present time. Policies denying genuine intellectual responsibilities to younger teachers would degrade the demands and attractions of the profession and would lead, in the long run, to serious deterioration of its standards and of the attainments of its members.

On turning from the intellectual to the economic aspects of recorded sequences, we may note that the question of proprietary interests takes on added urgency from the sheer magnitude of the investment. The fact, moreover, that a comparatively large outlay is required both for the production and for the use of recorded sequences may have two educationally undesirable consequences: first, a reluctance to change, thus tending to perpetuate the maladroit along with the praiseworthy (though new techniques permitting "erasure" and correction may alleviate this difficulty); and second, a compulsive tendency to use presentations merely because they are available and were costly, and operating costs are low, regardless of their detailed suitability for the specific purpose at hand.

We are more concerned, however, with the apprehension, apparently widely felt among teachers, that the use of recorded sequences may displace contact instruction to such an extent as to cause the demand for well-qualified teachers to decline seriously.

On the contrary, we believe, recorded sequences will lead to more rather than less demand for personal instruction. There is no prospect that the "average" student will be able to learn fully and efficiently without having some access to an agency that can respond to his individual needs. There is no prospect that any agency will surpass a human teacher in responsive capabilities. Except under very special conditions, recorded sequences seem unlikely to replace direct group instruction entirely.

The same rising enrollment, moreover, which lends special attraction to recorded sequences, does so in large part because of the growing shortage of qualified teachers. If our schools and colleges are to discharge their responsibilities effectively in coming years they will need to use every genuinely helpful teaching aid, including presentations, along with every qualified teacher who can be persuaded to serve under circumstances then prevailing.

The recorded sequence is the newest fruit of a new technology. Its ultimate effects cannot be reliably predicted, and the preceding tentative opinions are subject to adjustment in the light of accumulating experience. The principal question, perhaps, concerns the most advantageous distribution of responsibility, all things considered, between personal instruction, direct group instruction, and the newly feasible recorded sequence. The proposed Standing Committee

could exert a most beneficial influence in assuring that sound educational considerations are kept in the foreground as policies governing the use of presentations are developed.

NOTES ON THE STATUS AND FUTURE OF FILMS AND TV IN COLLEGIATE MATHEMATICS EDUCATION*

PHILLIP S. JONES, University of Michigan

The Committee on Instructional Films was appointed in September 1955 as a result of a suggestion to the Board of Governors of the Mathematical Association of America that the Association consider producing, for classroom use in colleges, a sequence of films based on an outstanding freshman course.

The Committee has (1) corresponded with itself and others; (2) interviewed producers of films and kinescopes, and persons at the Educational Television and Radio Programming Center; (3) viewed films; (4) consulted with persons from the Department of Audio-Visual Education of the National Education Association and from the Eastman Kodak Company; (5) consulted with other mathematicians; (6) studied various reports on the teaching effectiveness of films and television, and (7) met for discussions in Washington November 15–17, 1956 and in Ann Arbor July 18–21, 1957. These meetings were made possible by a grant of funds from the National Science Foundation.

The Committee found such an interest in films and television as teaching aids and such a rapidly changing situation as to experimentation with and use of these media that it submitted its report in two parts. Part I was its recommendations to the Board as to the activities in this area which it thought desirable for the Association to undertake. Part II, on which this note is based, was designed both to elaborate and to document the specific recommendations of Part I and also to summarize what seemed to it to be the present status and immediate future of (1) research in the use of these media; (2) current and recently completed projects using films and television in teaching mathematics; (3) the needs and possible audiences which it considered.

That there is an increasing number of mathematical television projects is revealed by the list in Appendix A. A survey of these projects will show that they can be roughly classified as: (1) "cultural," where the chief objective is

* This is a revised version of Part II of the report submitted to the Board of Governors of the Mathematical Association of America dated July 30, 1957 by its Committee on Instructional Films composed of Howard H. Campaigne, Marguerite Lehr, Phillip S. Jones (Chairman).

to interest a general audience, give it some idea of a small segment of mathematics, and perhaps by this device give it a better appreciation of what mathematics is, or at least of wherein some of its fascination lies; (2) "public relations," and/or "guidance," where various groups, but largely public schools, have tried to show taxpayers and parents how their children are taught and to interest students (and their parents) in electing more mathematics; (3) "course," where the object is to teach a definite body of mathematical knowledge, often, but not always, for high school or college credit.

If one does this classification on a chronological basis one perceives very clearly that whereas programs of types (1) and (2) predominated in the earlier years, the preponderance of mathematical television programs are now of type (3) and are frequently "for credit." The courses range from high school algebra taught for persons often out of school for some time who are working toward a high school diploma, through courses in solid geometry and trigonometry to make up college entrance deficiencies or for employed persons seeking extension courses, to full fledged college freshman courses, and to a course in calculus on color TV for the "in-service" training of secondary school teachers.

The Committee has done no research itself (although two of its members have had TV experience), and has not read all of the material on the subject available (Appendix B contains a rather randomly selected list of references). However, it does believe that the following conclusions are fairly valid. It is not clear to us, however, that all the results obtained from studies of teaching by films and TV can be automatically transferred to the teaching of mathematics. Further we are not certain that the tests which frequently show that students learn as well by television as with standard instruction have actually tested all of the important ideas, attitudes, insights which we hope we teach in addition to facts and techniques. Finally, in many statistical studies the differences between instructional groups were not very significant—a sort of negative endorsement of film and TV instruction. With these reservations we list the following conclusions.

1. A topnotch teacher can extend his effectiveness through the use of television (or films).

2. Pupils learn at least as much in television classes as with conventional instruction.

   (These two statements were attributed to Dr. Alvin C. Eurich in a Fund for the Advancement of Education press release, but they, especially the second, are supported by evidence in studies involving the teaching of the slide rule [1], general science, chemistry [30], army and navy skills [30, 41], where the numbers in square brackets refer to items in the bibliography of Appendix B).

3. In some cases pupils have learned more by television than by standard instruction, and in a few they learned less [39]. Able students profited more from high school algebra on TV than did slower ones [11], but the

army found TV more effective than live instruction for persons of low ability.

4. Disadvantages of television instruction are
   a. Lack of teacher-class discussion and interaction. (There are some devices to offset this—*e.g.*, two-way communication, discussion group follow-up of the film, devices built into the film such as asking questions, pauses for thought or discussion, repetitions of parts of the film or TV presentation.)
   b. Dislike of television instruction: Actually students frequently prefer it to other instruction, especially where they see demonstrations better on TV than in a lecture hall. On the other hand, a group of TV-instructed psychology students did substantially as well as others but *liked* the subject less well. In one study, superior students liked TV instruction less than the less able students, but preferred it to large classes.

5. Advantages claimed for film and TV instruction
   a. Good teachers and mathematicians on film may do a better job than poor teachers with a small class, or than good teachers in large classes.

      In this connection readers may be interested in the following comment from C. B. Read of the University of Wichita which has ***not*** tried teaching by television, "Prior to establishing instruction by television the University of Wichita decided to experiment with live teaching, using large classes and the lecture method. Conference hours were set up under the supervision of experienced faculty members (not graduate assistants) trying to duplicate as far as possible the situation which would be used with instruction by a lecturer over television. We have used three different professors in giving the lectures. The course has been restricted to intermediate algebra. After evaluating the results which have been tried for three semesters both the department and the University Administration is thoroughly convinced that this method of instruction is definitely inferior to our customary small-sized classes with personal contact by the teacher for each group. Our experience has been that under this plan students do not take advantage of conference hours to any great extent. More than 50 per cent of those enrolling in the course have failed to complete either through withdrawal or failure. For this reason the University has abandoned any attempt to utilize television or films as a method of teaching mathematics classes."
   b. Experts and famous scholars may be seen and heard in more places.
   c. Models, special devices, "animation" may be used to enrich instruction.
   d. Instructors viewing good films may see how to improve their own teaching.

e. Instructional time may be freed and made available for individual instruction and help, or for teaching more advanced courses.
f. The ease of repeated viewing of a filmed lesson may help adapt instruction to the slow or ill-prepared student.

6. Supplementary materials, outlines, reading lists, *etc.*, increase the effectiveness of film instruction, as do preliminary and follow-up discussions.

7. Kinescope film quality is a little more variable than that of regular movies and is never as good as a very superior movie, but is better than poor movie films and is very adequate for teaching purposes.

8. Planning, and sympathetic, intelligent, and cooperative direction including adequate walk-through rehearsals are essential to production of good films or kinescope. Were it possible, trial runs on closed circuit TV might help to produce better kinescopes.

In its discussions, particularly those held in Washington in which a number of additional persons participated, the Committee listed the following groups or "audiences" for which films might serve a useful and somewhat unique purpose and for which the Association and its members should have some real and immediate concern. (The committee report which stresses *films* more than *TV* is still significant here because kinescope films, made relatively inexpensively at the time a "live" TV program is produced, may then be available for classroom showing elsewhere or for repeated showings on television.) These audiences somewhat correspond to objectives for films. It is important that the objectives be clearly defined before embarking upon the production of TV programs or films.

A. *Audiences for single films or short film sequences.*

1. *Superior underclassmen*, persons who had been good high school students. Purposes: Enrichment of their course work, giving them an opportunity to see "a mind in action" embodied in a well-known mathematician-expositor who might use an approach different from standard texts, add supplementary materials, try to give them "insights" and "appreciations."
2. *Upperclassmen who are not necessarily mathematics majors*, but who are interested and able. Purposes: Similar to those of audience 1 but with material presented at a higher level of maturity.
3. *Upperclassmen, mathematics majors.*
4. *Secondary School Teachers*—especially those in the field who might be reached by extension courses and workshops.
5. *Adults* who are interested in continuing their "general" or "cultural" education.
6. *Superior high school students*—"enrichment" films.

7. *High School students, college freshmen and their parents*—"guidance" or "career" films.

B. *Audiences or populations for "course" films or kinescopes.*

1. *The general freshman course* where the objective of the film would be to conserve and extend the effectiveness of a teaching staff which is in short supply. The Committee does not believe that a film will ever be a better teacher than a good teacher with a small class. It may be better than a poor teacher or a good teacher in a large lecture hall. Washington University and several others have made beginnings here. (See Appendix A and the articles in this issue.)

2. *The class for really superior freshmen.* They need inspiration and contacts with persons and ideas not always available on all campuses, but their backgrounds vary and the number of schools which could and would offer a particular course for this group is limited, hence it is doubtful if TV or films for them is desirable except for the enrichment of regular instruction.

3. *A nontraditional first year course,* such as "Finite Mathematics," "Universal Mathematics," or "Allendoerfer and Oakley." Kinescopes for this might serve to "spread the gospel," but the problems of whose gospel would be accepted by whom seem to make such a project an uncertain place to begin experimentation in TV and film teaching.

4. *Analytic geometry and calculus.* The growing popularity of this combination as a freshman course makes it a good candidate for film.

5. *Trigonometry.* The Committee felt that there might be value in one or more film series in this area. Several schools have worked on this (see Appendix A). The objectives of this series might be (a) to improve the quality of teaching by making an outstanding teacher available to more students in this one course, and thereby at the same time improving instruction in other courses which would be better staffed as a result of the lightening of the elementary teaching load carried by experienced staff members; (b) to improve the quality of teaching by showing to all instructors an outstanding teacher at work (this could even be viewed by and have an effect upon high school teachers); (c) to improve the quality of teaching by showing the use of charts, graphs, oscilloscopes, and other devices such as animation; (d) to improve or modernize the content of courses now taught by giving a modern emphasis (or de-emphasis) to the solution of triangles, the role of angles, the nature of periodic functions. Some single films of such a series might be usable for enrichment purposes by persons not using the entire series; (e) to improve the "pace" of the course without handicapping the ill-prepared student who may get his added help from repeated viewings rather than from a slowed pace which may dull the interest of better students.

6. *College Algebra.*

7. *Analytic geometry.*

The problems of adaptation to local situations and texts suggest that local development of course films may be preferable to Association- or foundation-supported projects. Several schools have been working on this (See Appendix A).

Note that the Film Evaluation Board appointed by the Division of Mathematics of the National Academy of Sciences—National Research Council has recommended the establishment of a standing Committee on Mathematical Films and Television. The functions of this proposed committee and the policies recommended by the Board to producers of mathematical films and television programs are discussed in [12] in Appendix B.*

APPENDIX A—MATHEMATICS ON TELEVISION

(The Committee's Chairman has gathered this data from many sources, including newspaper accounts. He has written to every person or group mentioned, but has not had confirming data from all of them. There are probably inaccuracies and incompletenesses in what follows, but the interest in the topic seemed to him to justify the publication in spite of this.)

1958–1959 Projects
(Added in proof)

Martin T. Wechsler of Wayne State University, Detroit, is making 64 30-min. kinescopes to be used to teach *intermediate algebra and trigonometry* in the fall. This is intended for prospective freshmen with high school deficiencies, to be used on both closed-circuit and open broadcasts.

The University of Detroit will teach *analytic geometry and calculus* for 4 hours credit in 4 30-min. lectures per week in the fall.

Emil Berger presented 12 30-min. programs for in-school viewing by high school students over KCTA, St. Paul, Minnesota, early in 1958.

1957–1958 Projects

Station-Sponsor-Personnel	*Topic and/or Purpose-Length*
1. Closed circuit—Massachusetts Institute of Technology. Hartley Rogers, Jr.	Elective freshman course for nonmathematics majors. 6 1-hr. TV lectures. Remainder of course given in large lecture room.
2. Closed circuit—color. Walter Reed Army Medical Center, Washington, D. C. and University of Maryland, R. A. Good.†	*Foundations of Analysis.* For teachers. 26 50-min. lectures. Sponsored by National Academy of Science-National Research Council's Advisory Board on Education and A.A.A.S. Kinescopes and an evaluation supported by the Fund for the Advancement of Education.
3. Educational TV stations. N.B.C. and the Educational Television and Radio Programming Center.	13 30-min. programs planned by J. R. Newman in spring of 1957. Second series, fall, 1957 planned by Howard Fehr.
4. Closed circuit. Stephens College.	8 25-min. lectures by Adele Leonhardy followed by small-group student discussions.

5. The Fund for the Advancement of Education has given $986,000 to schools in Atlanta, Cin-

* See the article by F. A. Ficken in this issue, pp. 393–403. This committee has now been appointed. With C. B. Allendoerfer as chairman, it includes R. A. Good, G. A. Hedlund, P. S. Jones, E. J. McShane, J. R. Mayor, P. C. Rosenbloom.

† See this issue, pp. 426–427.

cinnati, Detroit, Miami, Norfolk, Oklahoma City, Philadelphia, Wichita, and the states of Nebraska, and Oklahoma to begin regular classroom instruction to large classes in September, 1957. Algebra is among the subjects to be taught to junior and senior high school students. There will be accompanying studies in methods of teaching large classes.

6. Alabama Educational Network.* WBIQ Birmingham, WTIQ Mumford, WAIQ Andalusia, University of Alabama.	*Mathematical Potpourri.* Popular lectures by various staff members, ½ hr. weekly, 16 weeks, fall. *Topics for high school students.* Designed for in-school viewing, fall and spring. 15 members of mathematics staff have participated.
7. Closed circuit. University of Alabama* Holland C. Filgo, Jr.	*Trigonometry.* 3 groups, TV and control. 30-min. lesson followed by 20-min. discussion.
8. WTTW. Chicago City Junior College, Jerome M. Sachs and Florence M. Miller.	*Introduction to Mathematics.* 3 hrs. credit, ½ hr, 3 days a week, 46 lessons. Live, spring 1957, by kinescope in fall of 1957. See Appendix B, [42], and projects **9, 36.**
9. WTTW. Chicago City Junior College, Jerome M. Sachs, Henry Patin.	*Slide Rule.* 1-hr. credit, ½ hr., 1 day a week, 14 lessons. Fall 1957 and spring 1958. Kinescoped.
10. WFBG-TV. Pennsylvania State University, Steven A. Adler.	*Topics in elementary algebra and recreational materials.* 27 ½-hr. programs.
11. Closed circuit. Pennsylvania State University, George N. Rancy, L. T. Dunlap, Isador M. Sheffer.	*Algebra for non-technical students.* 3 hrs. credit. 3 hrs. a week, 15 weeks, control classes, tutorial sessions, and talk-back system. Spring 1958.
12. WFBG-TV. Pennsylvania State University.	*Cultural Program.* 13 one-hr. sessions. Scheduled for summer 1958.
13. KUHT. University of Houston.†	*Trigonometry.* 27 lectures of 44 min. were filmed in summer of 1957 to be shown at rate of 2 a week on both open circuit TV (twice) and by projectors (three times). Students also required to attend a one-hour conference each week.
14. Kinescopes. Westinghouse Broadcasting Co.	*Adventures in Numbers and Space.* 9 ½-hr. programs devised and supervised by H. F. Fehr with script written by Bill and Cora Baird and using their puppets. To motivate junior high school students.
15. Kinescopes. University of Michigan, A. J. Lohwater.	*The Meaning of Geometry. Modern Geometry. The Element of Chance.* 3 ½-hr. programs distributed to a number of TV stations.
16. WEWS. Western Reserve University, Robert Carson.	*College Algebra.* 44 ½-hr. programs, 3 days a week. Credit. Scheduled for fall 1958.
17. University of New Mexico and Albuquerque Public Schools.	Kinescopes distributed to five schools pending completion of TV broadcast facilities.
18. WQED. Adult School of the Air, Pittsburgh, Harry A. Snyder, Director.	*General Mathematics and Business Arithmetic. Algebra* is scheduled for 1958–1959. This program began in 1954. See also projects **27, 28** and **51** listed below. Courses prepared for army and State of Pennsylvania exams leading to high school diplomas.

* See the article by Ayrlene McGahey Jones in this issue, pp. 421–424.

† See the article by Martin Wright in this issue, pp. 425–426.

19. Closed circuit. Purdue University,* John Dyer-Bennett, William R. Fuller.	*Freshman College Mathematics.* 5 hr. credit. 5 days a week. 30 min. on TV followed by 20-min. session with graduate assistant in each room.
20. KETC. St. Louis, Washington University,† H. M. MacNeille, H. Margaret Elliott, J. K. Goldhaber, I. I. Hirschman, Jr., Jack Indritz, Ross R. Middlemiss.	*College Algebra and Trigonometry.* 5 units. 45 min. 5 days a week on film. Twice daily on open circuit supplemented in first semester by non-TV lecture twice daily. Film only in second semester, twice daily on screen. *Analytic Geometry and Calculus.* 5 units. 45 min. 4 days a week, second semester; live on open circuit, once daily. Supplemented by two non-TV lectures daily. *Intermediate Algebra.* 2 units. 30 min. 3 days a week, live on open circuit, to supplement instruction for about 500 University College (evening) students who meet with an instructor one night a week. *Theory of Games.* One unit. 30 min. one day a week. (See project **33** also.)
21. KUON-TV. University of Nebraska, David Wells.	*General Mathematics.* *First Year Algebra* (High School). *High School Geometry.* 5-hr. high school credit per semester. 2, 3, and 5 TV presentations a week, respectively, each 30-min. long. Correspondence study is also involved, supported by the Fund for the Advancement of Education. Begun in 1956–1957.
22. WOI-TV, Ames, Iowa. Iowa State Teachers College, Cedar Falls, Irvin Brune.	*The Teaching of Arithmetic.* 10 90-min. programs for college credit.

1956–1957 Projects

Station-Sponsor-Personnel	*Topic and/or Purpose-Length*
23. WATV, Newark, N. J. Ernest R. Ranucci.	*Spotlight on Mathematics*, 6 30-min. popular- interest programs. See project **61.**
24. WTVS. Detroit Public Schools, Franklin Frey and Geraldine Dolan, Cass Technical High School.	30 min. on *conic sections* and their applications.
25. KCTS, Seattle. University of Washington, Carl B. Allendoerfer.‡	*Intermediate algebra.* 5 credits; 20 weeks, 30 min., 3 times a week.
26. KCTS, Seattle. University of Washington, Carl B. Allendoerfer.‡	*Plane trigonometry.* 3 credits. 3 ½-hr. programs a week, 10 weeks on new material, one reviewing home work.

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* See this issue, pp. 430–439.
† See this issue, pp. 440–443.
‡ See this issue, pp. 444–446.

27. WQED, Pittsburgh. Pittsburgh Public Schools, Alvin Stuart.	*Fifth-Grade Arithmetic.* 28 classes in 23 schools, 20 min. on TV 5 days a week with 20-min. classroom instruction. 175 school days. Fund for the Advancement of Education. See Appendix B **[10]**.
28. WQED, Pittsburgh. Jack Roush.	36 ½-hr. kinescopes on *algebra* were made under an Armed Forces contract for use by the United States Armed Forces Institute. See projects **18, 51** also.
29. Closed circuit.* Purdue University, John Dyer-Bennet, Merrill E. Shanks.	*Calculus.* 4 hr. credit. 3 hrs. a week on TV, 1 hr. conventional instruction.
30. Closed circuit. Mount Pleasant High School, Schenectady, N. Y. Edward Sherley.	*Intermediate Algebra and Trigonometry.* 2 classes. Supported by the Fund for the Advancement of Education.
31. KETA-TV, Oklahoma City.	*High School mathematics.*
32. KETA-TV, Oklahoma City. University of Oklahoma, Arthur Bernhart.	*Intermediate Algebra.* 3 hrs. college credit. Three ½-hr. lessons a week for 18 weeks.
33. KETC, St. Louis. Washington University,† Ross Middlemis, Holbrook MacNeille, Margaret Elliott.	*College algebra and trigonometry.* 5 credits. 45 min., 5 days a week, twice on open and 3 times on closed circuit. See project **20** also.
34. WTTV. Indiana University, Philip Peak.	*High school trigonometry.* ½ unit. 18 30-min. shows to provide the course for small high schools.
35. KUON-TV. University of Nebraska, David Wells.	*Beginning high school algebra.* Credit. 20 min. a day, accompanying correspondence study material. Fund for the Advancement of Education.
36. WTTW. Chicago Public Schools, Jerome Sachs.	*Ten experimental lessons on quadratics for high school students.* See Appendix B, **[11]**. See project **8.**
37. Alabama Educational Network.‡ University of Alabama, Ayrlene Jones, Susie Lee Ward.	*College Preparatory Algebra.* No credit. 12 weeks, 30-min. lessons. Summer 1956 and 1957.
Ayrlene Jones.	*Trigonometry.* Credit. 16 weeks, 30-min. lessons. Fall.
Ayrlene Jones.	*College Algebra.* 16 weeks, 30-min. lessons. Spring. *Mathematical Potpourri.* Popular lecture by various staff members. 15 weeks.
38. Hagerstown, Md.	*Plane geometry.*

1955 Projects

Station-Sponsor-Personnel	*Topic and/or Purpose-Length*
39. WHA. University of Wisconsin, Professor Sara Rhue.	*Arithmetic.*

* See this issue, pp. 430–439.
† See this issue, pp. 440–443.
‡ See this issue, pp. 421–424.

40. KUHT. University of Houston, School of Business Administration, Rolland Crouch. — *Mathematics of Finance.* 3 credits, 45 min. twice (?) a week.

41. KTTS-TV, Springfield, Mo. Springfield Public Schools, Robert C. Glazier. — 2 *arithmetic demonstration lessons.* 30 min. each. 2 *Why Mathematics?* 30 min. each. See Appendix B, [10].

42. WCET, Cincinnati. E. Havlish, Cincinnati Milling Machine Co., and public school students, arranged by Mildred Keiffer. — *Measurement in Industry.* 3 25-min. programs designed for junior and senior high school classes to show the importance of mathematics in industry.

43. WOI-TV, Ames, Iowa. Iowa State Teachers College, Cedar Falls, Irvin Brune. — *The Teaching of Arithmetic.* 10 90-min. programs for college credit. See Appendix B, [10].

44. WKAR-TV, East Lansing. Michigan State University, B. M. Stewart. — *Nomography and graphical methods.* 12 30-min. programs.

45. WQED, Pittsburgh. Pittsburgh Public Schools, Mrs. Anita Seewald. — *Fifth Grade Arithmetic.* 20 classes in 10 schools. TV teaching for 25 min. of the 40-min. arithmetic classes. 5 days a week for 178 days. Supported by Fund for the Advancement of Education. See Appendix B, [10].

46. WUNC-TV, Raleigh, N. C. North Carolina State College, Henry C. Cooke. — *Solid geometry.* Taught in summer. Noncredit but to fulfill college entrance requirements. 30 min. once a week, 16 weeks. This has been presented three times as of February 1958.

47. WRC, Washington, D. C. Louise R. Grover, Faith F. Novinger, Carl Hansen, pupils from public schools. — *Reading, Riting and Rithmetic on the High School Level.* 2 15-min. programs to show modern teaching techniques.

1954 Projects

Station-Sponsor-Personnel — *Topic and/or Purpose-Length*

48. WKRC-TV, Cincinnati. Teachers, students, Mildred Keiffer, Supervisor, Mathematics 7–12. — *Demonstration lessons for N.C.T.M. meeting.* 3 30-min. programs. Twelfth, eighth, fifth, and second grades. See Appendix B [8].

49. Kinescopes. University of Michigan, Phillip S. Jones. — *Understanding Numbers.* 7 30-min. kinescopes made for The Educational Television and Radio Programming Center and widely distributed by them. Available for audio-visual use from Indiana University Audio-Visual Center. See also projects **63, 67,** and Appendix B, [4], [5].

50. WREX-TV, Rockford, Ill. Northern Illinois University, DeKalb, Ill. Herbert Miller. — *Magic Squares.* One 15-min. program. Appendix B, [9].

51. WQED, Pittsburgh. Jack Roush. — *High School Algebra.* In preparation for state high school credit examinations—18 lessons, 30 min. each. See also project **18.**

52. KETC, St. Louis. Panel discussion: Dr. Harold Fawcett and Mr. Eugene P. Smith of Ohio State University and students from Ohio and Missouri high schools, arranged by Dr. Margaret Willerding. — *Should I Study Mathematics?* One 25-min. kinescope. Originally made in connection with the Dec. 1954 meeting of the N.C.T.M. in St. Louis. This is program 1 in Series 1 of "High Time, a Panel Discussion Program for High School Students," designed for in-school viewing and accompanied by a published "High Time Study Guide."

53. Seattle, Washington. — *Demonstration classes* for public relation purposes.

54. WGAL-TV, Lancaster, Pa. Elizabethtown College, Carl Heilman. — 4 30-min. programs: Contributions of Pythagoras, Fermat, and Pascal, Descartes, and an Introduction to Nomographs.

55. WFIL, Philadelphia, Pa. Bryn Mawr College, Marguerite Lehr. — *Invitation to Mathematics.* 15 27-min. programs. *E.g.*, "Regular Patterns and Symmetry," "Regular Shapes," "Products and Primes," "Matching and Counting, Big and Infinite," "Maps, Terrestrial and Otherwise," "On Measure of How Likely," "Questions on Least and Most." See Appendix B, [6].

56. WTAV, Newark. Rutgers University, Fred G. Fender. — *This is Mathematics.* 13 programs. Number systems and number theory, greatest and least, transfinites. See Appendix B, [7].

57. Marquette University. William A. Golomski, H. P. Pettit. — *Statistics, Geometry.*

1953 Projects

Station-Sponsor-Personnel	*Topic and/or Purpose-Length*
58. Illinois Institute of Technology. Karl Menger.	Two programs.
59. WGAL-TV, Lancaster, Pa. George R. Anderson.	*Let's Look at Mathematics.* 16 programs. Appendix B, [1], [2].
60. Texas State College for Women. Harlan C. Miller.	One noon-hour show—Moebius strip, *etc.*
61. WATV, Newark. Ernest R. Ranucci.	*Spotlight on Mathematics.* 4 30-min. programs. See also project 23.

1952 Projects

Station-Sponsor-Personnel	*Topic and/or Purpose-Length*
62. WBEN-TV, Buffalo, N.Y. Buffalo teachers, 10–14 students, commentator Louis F. Scholl, Director of Mathematics.	*Demonstration lessons.* Chiefly grades 1–8. Public relations. 30 min. See Appendix B, [3], [14]. These lessons were continued through 1955–1956.
63. WWJ-TV, Detroit, Michigan. Phillip S. Jones, University of Michigan.	*Understanding Numbers: Their History and Use.* 7 18-min. programs, "The Earliest Numbers," "Bases and Places," "Big Numbers," "Fundamental Operations," "Short Cuts and Computing Devices," "Fractions," "New Numbers." See also project 49 and Appendix B, [4], [5].

64. WOW-TV, Omaha. Creighton University, John Englund.
Counting—Then and Now. 15-min. program. Also shown over this station in 1955 and over KOLN-TV in 1954. See Appendix B, [24].

65. WGAL, Lancaster, Pa. State Teachers College, George R. Anderson.
The Slide Rule. 6 30-min. programs. See Appendix B, [1], [2] and program 68.

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Visual Devices in Mathematics.
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66. WOOD, Grand Rapids, Mich., University of Michigan, Phillip S. Jones.
Mathematics Around Us: I Numbers, II Geometry (Conics, Parallelograms). Two 27-min. programs.

1951 Project

Station-Sponsor-Personnel	*Topic and/or Purpose-Length*
67. WEWS, Cleveland, Ohio. Cleveland teachers, 8–10 students. Commentator. Herschel Grime, Directing Supervisor of Mathematics.	*Demonstration lessons.* Primary, upper elementary, grade 7 (Mental Arithmetic), grade 8 (Informal Geometry), grade 10 (Plane Geometry, Micrometer), grade 10 (Shop Mathematics Class, slide rule in a Trigonometry class). 15-min. programs. 2 each year to the present time.

1949 Project

Station-Sponsor-Personnel	*Topic and/or Purpose-Length*
68. WGAL-TV, Lancaster, Pa. State Teachers College, George R. Anderson.	*The Slide Rule.* Six 15-min. programs. See Appendix B, [1], [2].

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SPIES, ELECTRIC CHAIRS, AND HOUSEWIVES

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In this article I write of my personal experiences arising from three television shows given on WFIL-TV, Philadelphia, January 8, 15, 22, 1957. Whereas many of the papers in this issue of the MONTHLY are devoted to various campus studies and are statistical in nature, my experience differed considerably. Three reasons for writing this chronicle are: (a) to acquaint the readers of the MONTHLY with my activity and thereby encourage them to enter this field; (b) to set forth a plan for a use of television in mathematical promotion; and (c) to learn from those who may respond to this article and therefore benefit doubly. The title of this paper will be explained presently.

Professor Francis A. C. Sevier, lately chairman of the mathematics department, College of South Jersey (Camden), having successfully presented a series of fifteen-minute programs over WFIL-TV, 1955–1956, was asked to supervise, for the academic term 1956–1957, that station's "University of the Air Series: A Survey of the Sciences." Throughout the semester guest speakers discussed topics in a variety of physical sciences and it was my lot to conclude the series with three talks on mathematics.

During the summer of 1956 I prepared scripts for the three talks. My subject was to be "Mathematics in Our Western World" wherein I hoped to talk about "The History and Development of Western Mathematics," "Mathematics and Human Institutions," and "Mathematics *Is* Fun."

Early in September, the participants in WFIL-TV's "Series" were assembled at the studios to be briefed on the mechanics of the show. We were told that the "University of the Air" had been telecast for seven years; that between 9 and 10 A.M. each weekday morning three "classes" are held; that the 9:20–9:45 class "A Survey of the Sciences" was chosen for rebroadcast by kinescope films

* Science Faculty Fellow, National Science Foundation.

over affiliated stations: WFBG-TV (Altoona), WNBF-TV (Binghampton), WNHC-TV (New Haven), and WLBR-TV (Lebanon); that approximately two or three million people would view the class (at least at the outset—this was *our* challenge!). This last bit of information caused me some personal discomfort.

Specific instructions dealt with what type of stage properties could be used (slides, charts, models, film strips, blackboards, *etc.*), dress, and stage presence. With respect to the last item, the director evidently felt that a natural, unpretentious attitude was necessary if we were to hold our audience. To this end, we were given no further rehearsals, makeup tests, or plan of action regarding camera movement. The director emphasized that unless informality prevailed, the viewer could instantly replace us with a mere twist of the wrist. Since I was to appear four months later, I promptly prepared strenuously for my "extemporaneous" talks. In this respect I was perhaps more fortunate than the others, who, lacking adequate time, resorted mainly to reading from prepared texts.

At this point I should like to indicate my aim in presenting such an austere sequence. Briefly, I hold that mathematics and the current well-known situation in mathematical education (let's face it, I mean apathy on the part of students and public in general) can be improved if a well-planned promotional scheme can be brought to fruition. I feel that if mathematics—its history, its development, its *raison d'être*—could be properly presented to parents of high school students, say, then a more friendly attitude towards the subject would develop in the home. Any attempt to "sell" mathematics to students during school hours will fail if the after-school sentiment at home is ignored. I hope to awaken a interest by showing the adult lay public that mathematicians are human, that work in mathematics can be rewarding and gratifying, and that hard work and willingness rather than genius are the necessary ingredients. With this awareness, perhaps parents and teachers will be able to motivate the new generation of students.

At any rate, with this hypothesis in mind, I gave my first talk on "The History and Development of Western Mathematics." I spoke about the "reckoning" phase of early mathematics, the specific knowledge of Egyptian society and the "new" concepts introduced by Greece. I attempted to trace the threads of the mathematical cloth through the dark ages, the Renaissance, and classical times. From *Scripta Mathematica* I obtained the well-known portfolio of portraits which were viewed at appropriate intervals (thus I hoped to add some interest to an otherwise lengthy talk).

(Part of this paper's title may be explained at this point. As an illustration of the sort of thinking sometimes involved in mathematics, I posed the following problem (referring to a picture which showed a man, question mark over his head, gazing at two identical electric chairs behind which stood two executioners):

Unless this man can ask a suitable question and act properly, he is doomed;

for one of these chairs is wired while the other isn't. Of course, the executioners know which it is. Also, one of these men is a truth-teller and the other a persistent liar. Again, each executioner knows who is the liar. The victim is allowed to ask of either man (but not both) just one question whose answer is either "yes" or "no." What question should he ask?

I received an unexpected volume of mail in response to this puzzler and to an offer to send a bibliography and other material. I'll discuss this later.)

Of course, without a rehearsal, it was bound to happen: some portraits did not agree with the individuals under discussion. Throughout the talk, the emphasis was on the men and ideas rather than the techniques developed (I hoped to show in the second talk how the rigor and beauty of mathematical thinking affected men's ideas in politics, economics, art, literature, and music). Unfortunately time ran out before I could demonstrate that twentieth century mathematics exists and is flourishing!

Two points ought to be made now: rehearsals are necessary to get the timing set perfectly (I needed two minutes to run out of material); it is too ambitious to reduce twenty-five centuries of history to twenty-five minutes. Possibly, in talking with professionals in this field, I would have come to realize this.

Unexpectedly, mail came in to the studio which reflected a mild interest in what I had said. However, it was suggested that I review my scripts with an eye to toning down the level. To this end I decided to discuss mathematics and art (demonstrating with the use of slides from the Philadelphia Museum of Art the idea of Greek simplicity, Latin ornateness, pre-renaissance "flat" paintings, perspective and the development of "depth," and finally the ultra-modern movement), mosaics and line drawings (illustrated by the well-known charts of Colonel R. Beard), mathematical influence in architecture (I showed photographs of the Hotel Waikiki with its hyperbolic paraboloidal roof), and mathematics and music. To demonstrate simple applications of Fourier's work, I prepared on tape, recordings of various musical instruments and combinations. An oscilloscope was used to present sound visually. Various harmonic syntheses were performed with the purpose of demonstrating the usual elementary notions.

All this was planned to replace "Mathematics and Human Institutions" since I was told (secretly) to think of it now as "The Kindergarten of the Air Series." Now, about five minutes before going on, I was informed that only four of the dozen and a half slides could be used (the slides, being in color, wouldn't be telecast successfully, I was told)! More evidence that rehearsals are necessary, in spite of the admonition to be "natural."

I found that it was not too difficult to ad-lib half a program; but I would have preferred more advanced notice. Certainly the program content was out of balance. Generally, the views of the oscilloscope were poor, there being insufficient contrast on the 'scope face. Significantly, Professor Sevier, who introduced me each time and concluded each performance, noted publicly that that was the first class that ended "on time."

Regarding this hastily revised second program, I feel that the viewing public showed more interest in the light of its response and the general sentiment expressed by professional TV men. At any rate I decided to conclude the series as planned, demonstrating that "Mathematics *Is* Fun."

Once again poor program planning interfered with my plans. Just before going on, I was told to cut short my talk by five minutes in order that concluding credit announcements might be made and the moderator for the succeeding series might be introduced to outline his forthcoming fifteen week series. In this last program I discussed a few elementary algebraic puzzles, the Mobius strip with some demonstrations, another elementary topological pleasantry, and some paradoxes. I hope I astonished some people by calculating the thickness of a sheet of paper .001 inch thick folded in half fifty times. Some of Colonel Beard's art work was shown and explained and wax-paper folding exercises were demonstrated. The wax-paper creases came through with brilliant clarity, surprising the studio engineers. All told it was (for me at least) a pleasant romp through the field of mathematical recreation. Since many of the "answers" to the electric chair problem received during the past two weeks were incorrect, I repeated the problem and pointed out the inadequacy of such simple answers as "ask either executioner if he is the liar," "ask either if his chair is wired," *etc.* Time, however, did not permit my giving suitable answers.

I tried to avoid giving the impression in this last talk that mathematics is "just a collection of tricks," but does in fact have its lighter side. It was an attempt on my part to motivate an interest in mathematics by way of its entertainment value. The mail indicated that to a limited degree I succeeded.

This, then, is a chronicle of some seventy minutes I spent on television talking mathematics to an "adult" audience. To this of course was an aftermath: my "fan" mail. I received many letters and cards from people of diverse backgrounds ranging from truck drivers who "caught the show" in diners to housewives who were "relaxing after sending the children to school"—explaining another part of this paper's title. Most of the letters contained answers to the electric chair problem. Almost all letters requested the bibliography and more information about recreational mathematics. A few offered suggestions for improvement. Several hoped that the math series might be continued.

The following illustrations should be self-explanatory:

"Thank you for having thrown a new *light* on mathematics. I would appreciate your sending me further information on 'mathematical curiosities.'" (Philadelphia housewife).

"University of the Air is doing an excellent job and I want to thank all of the participants." (Philadelphia homemaker).

"Would it be possible to obtain a leaflet or paper listing some of items for a Girl Scout troop?" (Ventnor, New Jersey scoutmaster).

"Would asking one of the men to try the chair out first solve the problem? Could you send me a copy of some of the mathematical designs to use for scout work (girls) . . . I thoroughly enjoyed your program." (Upper Darby, Pennsylvania mother).

"Thank you for the mathematical puzzles. My husband and I are both brain weary after attempting to solve them." (Philadelphia).

"... I would ask the men to sit on the chairs. The man who refuses to sit down is standing behind the wired chair. Am I right ... I am curious and interested." (Kutztown, Pennsylvania woman).

"... My husband would like the solution to the electrocution puzzle. I am enclosing a self-addressed stamped envelope. Thank you for the enjoyable minutes we spent while you were on the air." (Gibbstown, New Jersey).

"... I have a son twelve years old who enjoys mathematics and would appreciate your sending him some mathematical quizzes." (Philadelphia).

"Please send me the information about geometric math puzzles." (Seventh grade teacher, New Jersey State Teachers College).

"I enjoyed the last lecture of the Survey of the Sciences (and wish there were to be more of the same type later). Is there a book I can get on the subject discussed?" (Moorestown, New Jersey viewer).

"... Thank you for the information and entertainment your program has provided." (Glenside, Pennsylvania boy).

"... Will you please send me these designs (Col. Beard's). I am an amateur artist and I teach children in my home. These children would enjoy drawing these designs." (Collegeville, Pennsylvania woman).

"If I asked you the following question would your answer be 'yes.' Is your chair the one that is wired?" (Engineer at WFIL-TV studio—devised during show). Explanation of solution and correct procedure outlined.

I have quoted some of the letters received to show what interest the recreational approach can achieve. Purists may snicker at this view of "mathematics," but a promotions man is more interested in ends, not means. Interest *was* evoked. Parents *are* becoming interested. How long will it be before their children are infected?

One or two other points ought to be made now. Many solutions to the electric chair problem depended upon elaborate devices (voltmeters, cut-out switches, *etc.*) and overlooked the point entirely. Some letters followed the pattern "... I didn't get the folding of the paper that would measure to miles and miles, but the figures are tricky and perhaps my hubby and son would enjoy trying to make some of them (in fact I know they will). So sorry I can't figure out the problem of which chair is wired and what one isn't. But I do believe the one who pulls the switch isn't a good Christian." (Holmes, Pennsylvania mother.) Several other letters posed problems for *me* to answer! Most gratifying were the replies that said essentially:

"... It seems to me that the prisoner could choose the non-lethal chair by asking the question: does the liar have the wired chair?" (Riverton, New Jersey housewife). This lady went on to give a neat and complete solution indicating her reasoning graphically and logically. She concludes by writing "If this is the correct answer to the problem, the credit goes to Dr. Sevier, who can teach anyone mathematics—even housewives." Can more be said?

Though it took time, I replied to each letter. *Scripta Mathematica* was sent a mailing list of people interested in knowing more about the philosophical and recreational sides of mathematics. With each response, aside from bibliography, I included an additional set of posers (without answers), one of which, to explain the last item in this article's title, was the following: A spy was

caught behind enemy lines one Sunday morning. Placed in a cell, he was told that late one night he would be executed in his sleep—in fact he would never see the following Sunday afternoon. Moreover, he would never know which evening was to be his last. Question: On which night will he die?

I received almost as many letters in response to this latter set of puzzlers and in particular in reply to the spy question as I did originally. Thus my correspondence did not drop off until the end of the second cycle.

Very significant, I have been told, is the fact that no derogatory letters were received. It appears to be well known to workers in the field that the ratio of negative to positive mail is enormous. Here then is a situation that contradicts the usual expectation.

This narrative then describes my experiences as an amateur television educator. In trying to stimulate an awareness of mathematics in the minds of adult laymen, an area of utmost importance if we are to be successful in motivating the young students of today and tomorrow, I have made an attempt. I hope I have convinced some of the readers that this is a worthy goal and that television can be used successfully as a means for bridging the gulf between the academicians and the laity. That experience in television acting is necessary to me now seems trivially obvious. Perhaps, as others have suggested, scripts should be written by mathematicians with professional actors employed to perform. I feel that with sufficient rehearsals, however, non-professionals can do a good job.

That care is necessary in preparing scripts is quite clear. With competent advice from others, I would never have put on that first telecast. I prefer a historical to a topical matrix in which to delineate mathematics. But in truth one must realize that each program, independent of the others, must be self-contained and concentrate on a single item.

It is true that such a series as mine is not altogether unique in its wide-spread audience, yet most of the current work in this area lay in closed-circuit, educational broadcasts for and by academic people. I invite others to join those of us who are currently trying to promote mathematics over open television. The kinescopes of my three talks are available on loan for any reader who wishes to view them. I would appreciate in return having the opinions and suggestions of those who do see them.

TELEVISION ACTIVITY, DEPARTMENT OF MATHEMATICS, UNIVERSITY OF ALABAMA

AYRLENE McGAHEY JONES, University of Alabama

The Department of Mathematics of the University of Alabama has maintained a wide and varied program of television activity since it began its first series of telecasts in the summer of 1956. This program has included a series of

popular lectures on several topics in mathematics, review courses in high school mathematics, college courses in algebra and trigonometry, an experiment in teaching trigonometry by closed-circuit television and a series of topics in high school mathematics for enrichment. With few exceptions all members of the department have participated in these programs. Not only has the medium been used to reach larger and more varied groups than the campus classroom allows, but it has afforded an opportunity of using and experimenting with visual aids which are not usually available for classroom use.

Alabama Educational Television Network. All of the programs and courses offered in mathematics by the University of Alabama (except the closed-circuit campus experiment) have been broadcast over the Alabama Educational Television Network consisting of three transmitting stations: WTIQ, Munford, Channel 7; WBIQ, Birmingham, Channel 10; and WAIQ, Andalusia, Channel 2. These stations, comprising the first educational television network in America, are owned by the state of Alabama, operated by the Alabama Educational Television Commission and reach approximately 80% of the state's population. There are four program agencies under contract with the Commission: University of Alabama, Alabama Polytechnic Institute, Alabama College and Birmingham Area Educational Television, Incorporated. Studios are linked in a statewide network via micro-wave facilities maintained by the Commission.

Popular lectures. This series of programs on various aspects of mathematics was planned primarily for gifted high school students and their teachers, college students and the general public. Fifteen members of the department participated and discussed such topics as: "Fallacies and Paradoxes," "Concepts of Infinity," "Electronic Computers and the Binary Number System," "Voting Power," "Zero is Not Nothing," "Orbital Missiles," "Alabama Paradox," "Number Congruences," "Professional Opportunities in Mathematics," "Games and Recreations," "What is Mathematics?" and others.* Since this series was well received during the spring semester of 1956–57 it was continued throughout the fall semester of 1957–58. The series was called *Mathematical Potpourri* and was telecast weekly.

College courses. Two departmental courses, college algebra and trigonometry, were broadcast on the network during the fall and spring semesters of 1956–57. They were also placed on campus closed-circuit to afford an additional opportunity of hearing lectures for those taking the courses on campus and others wishing to review the material. Trigonometry was offered for credit to qualified viewers off campus but only auditors registered for the course. These received outlines and lecture notes. Teachers in the several colleges and high schools in Alabama (and one across the line in Georgia) have indicated that some of their students supplemented their own courses in trigonometry and

* A complete list of topics with abstracts is available upon request. Kinescopes of several of these programs are also available for loan.

algebra with these telecasts. Also, many students enrolling as freshmen at the University have stated that they followed the courses at home. These were broadcast three times each week during the evening.

Closed-circuit experiment in teaching trigonometry. This experiment involves one control class and two television classes. The television teacher lectures approximately thirty minutes and graduate students conduct discussion of the lecture for the remainder of the period. Items under study include: the role of the graduate student in instruction by television and the effectiveness of this type of teaching apprenticeship, and the feasibility of using this medium to insure the instruction of large numbers of freshmen by experienced members of the department. It is expected that an evaluation of the experiment which is being conducted during the spring semester of 1957–58 will be made during the summer of 1958. This experiment is a part of the Study and Planning Program of the University of Alabama financed by a Carnegie grant.

Review courses in high school mathematics. During the summers of 1956 and 1957 a course in college preparatory mathematics was offered three times each week. The course was planned primarily for high school graduates preparing to enter college, for high school students continuing to study mathematics and for mature people wishing to review or study mathematics as a part of their general education. Arrangements were made for students to report to various centers throughout the state at the conclusion of the telecourse to take entrance examinations or examinations to remove high school deficiencies in mathematics. An outline of the course and problems were sent in installments, without charge, to all who requested them. Mail from throughout the state indicated an interest in the course by people of varied backgrounds and occupations, *e.g.*, a laboratory technician, a doctor, a housewife, parents and a teen-age boy. Some sent in homework papers to be corrected.

High school enrichment programs. Weekly throughout the academic year 1957–58 various members of the department are lecturing on topics selected by high school teachers in the areas of geometry, algebra and general mathematics. These are a part of the In-School Program offered by the Alabama Educational Television Network and are telecast during school hours. Topics include: "Transition from Arithmetic to Algebra," "Fundamentals of Geometry," "Problem Solving," "Compound Interest," "Functions and Graphs," and others.* Surveys indicate that 230 schools in the state are viewing some of these in-school programs.

Organization and production. The departmental television committee consists of the fifteen members participating in the various television series with the committee chairman acting as coordinator of all television activity and pro-

* The study guide for these programs is available on request. Kinescopes of some of these lectures are also available for loan.

ducer of the various programs.

Since the Broadcasting Services of the University include a complete workshop with personnel interested in creating new visual materials, it has been possible to experiment with many kinds of production aids. This has resulted in greater interest and pleasure in working with this new medium.

Tentative evaluation. Members of the department are unanimous in the opinion that teaching techniques are improved as a result of the careful planning television requires; also, one becomes increasingly aware of his students when they are replaced by the red eye of the television camera.

There is a strong feeling that television is best used for enrichment or as a supplement to conventionally-taught courses rather than a substitute for them.

Professor Julian D. Mancill, Head of the Department of Mathematics, states: "We are certain that television presents a challenging new medium of education and that it will experience a rapid development. However, we are just as certain that it will not, since it cannot, replace the classroom teacher in the sciences and especially in mathematics. There seems to be an unlimited potential for educational television in the areas of continuation education; enrichment of traditional courses through access to outside experts, animation and art; and general informational type of instruction."

THE GEOMETRY OF FOUR DIMENSIONS

A. ZAWROTSKY, Universidad de los Andes, Mérida, Venezuela

The following is a brief description of a film* that was made in order to facilitate the visualization of the geometry of four dimensions.

A tessaract was chosen as a representative polytope of four dimensions. The film represents 988 successive intersections of a tessaract by parallel hyperplanes (each photographed three times). The hyperplanes, however, are not perpendicular to a main diagonal, but form with any set of four concurrent edges of the polytope, angles of

$$\sin^{-1} 0.384, \quad \sin^{-1} 0.48, \quad \sin^{-1} 0.512, \quad \sin^{-1} 0.6,$$

respectively. This method, different from that of Dunchian described in Coxeter's well-known book *Regular Polytopes*, was chosen in order to give the pictures greater variety.

* This film has been offered for exhibition at the International Congress of Mathematicians to be held in Edinburgh, August 14–21, 1958.

A NEW PLAN FOR INSTRUCTING LARGE CLASSES IN BASIC MATHEMATICS

MARTIN WRIGHT, University of Houston

In January of 1957 the Mathematics Department of the University of Houston began preliminary study of an experiment designed to attack the problem of teaching an increasing number of freshman mathematics students without a corresponding increase in faculty and physical facilities while maintaining high academic standards in these basic mathematics courses. All known plans of instruction at other Universities that utilized video in any form were investigated before a decision was made on the details of the proposed experiment. Plane trigonometry was selected as the subject to be taught because of the large number of students enrolled therein. The main features of the plan used in the fall semester of 1957 to teach plane trigonometry to 350 freshmen students are listed below:

1. The course was divided into twenty-seven lectures of forty-four minutes each. All lectures were carefully checked by the Mathematics Department and filmed by the University Film Center. Six members of the Faculty participated as lecturers while others contributed as critics.

2. Lectures were given to the students at the rate of two per week. Each film was shown twice over open circuit television (morning and evening) and three times by projectors in viewing rooms of the Audio Visual Center (afternoons and Saturday mornings). Thus a student was given an opportunity to view each lecture several times if necessary to master the theory and problems.

3. All students enrolled for credit were supplied with a television supplement which contained routine instructions for the course, assignments, additional explanations and *an incomplete set of notes on all lectures*. The student completed the notes in this supplement as he viewed the lectures.

4. All credit students were divided into sections of 25–36 students and each section was required to meet one hour per week with a regular member of the Mathematics Department. These conference sessions were used to discuss lectures that had been viewed recently by the students, to answer any questions arising from these lectures, to distribute and collect homework assignments, and to review for examinations.

5. Ten hours per week of voluntary help sessions were scheduled in the afternoons and evenings, with a student assistant in charge, to answer questions about the solution of specific problems.

6. The examination schedule included two major examinations (midterm and final) and two minor examinations. The major examinations were two hours long and were given at an open period to the entire group. The

minor examinations were one hour long and were given in the weekly conference sessions.

Each major examination was preceded by a live television review, the material for the reviews being obtained from questions most frequently asked in conference sessions. Practice tests were distributed to students and discussed in weekly conference sessions prior to all examinations.

This plan of instruction deviated so radically from conventional procedure that an extensive orientation program was necessary for both students and faculty counsellors in order to assure a smooth operation.

Three hundred forty-three students participated in this program in the fall semester of 1957 and three hundred ninety-four are presently enrolled. No conclusion can be drawn at the present time as to the effectiveness of this plan of instruction. However, data is being collected for an evaluation of the program.

CLOSED-CIRCUIT COLOR TELEVISION

RICHARD A. GOOD, University of Maryland

The University of Maryland and the Advisory Board on Education of the National Academy of Sciences announce the experimental use of color television in the teaching of a graduate course, Foundations of Analysis, offered by the University. The closed-circuit color facilities at the Television Division of Walter Reed Army Medical Center are employed in teaching this course to a group of inservice high-school teachers of mathematics and science in the Washington, D. C. area. Associate Professor R. A. Good of the University is lecturer for the course and Dr. J. R. Mayor, Director of Education, AAAS, is serving as consultant. A grant from the Fund for the Advancement of Education of the Ford Foundation has made possible the production of color kinescopes which will be used in the comparison of various techniques. These kinescopes will later be made available to other suitably equipped institutions for further evaluation tests.

Twenty-six weekly lectures of approximately 50 minutes each (usually rehearsed at least twice on camera) are presented via television. About every third week the instructor meets the students in a regular class for examination, evaluation, recitation, or answering student questions. The course, carrying three credits, is still (at the time of this writing) in progress. The class is small in size. For these two reasons no significant evaluation of the project is presented

here. An independent agency will continue the evaluation during several classes via kinescope proposed for the 1958 summer session and next year.

The course stresses modern ideas concerning the background and basic concepts of the calculus. Set, relation (transitive, order, and many other kinds), function (as a type of relation), partition, continuity (defined by neighborhoods), limit, derivative, integral are highlights in the syllabus. To graphically portray relations, we have studied the fundamental notions of analytic geometry. Thus the course embraces more than just calculus concepts. In order to devote full justice to foundations, physical applications are not emphasized.

A major contribution to the teaching efficiency is being made by the very cooperative staff of the Television Division, particularly in connection with the art work. Many graphs, diagrams, charts are prepared in advance. Instead of the customary blackboard, we currently use colored chalk on a continuous roll of heavy paper which can be turned forward or backward. Whenever suitable, we use peg board, flannel board, magnetic blackboard, teleprompter (on camera, for lengthy definitions or statements of theorems). Color affords an excellent method to contrast, to emphasize, to distinguish. During a recent graphical portrayal of ϵ and δ, three studio assistants were employed. On a cardboard of unobtrusive color, dark coordinate axes and a brightly colored curve represented the function. Colored light from a projector gave a horizontal band 2ϵ units wide; variable shutters permitted ϵ to be arbitrary. Another projector with movable shutters beamed a vertical light zone of appropriate width 2δ. The third assistant could, whenever needed, overlay the graph with a transparent sheet having a vertical black-out line. We continue to experiment with any reasonable idea for making more understandable the delicate nature of our subject matter.

TV AND FILMS AT MONTCLAIR

B. E. MESERVE AND E. M. MALETSKY, State Teachers College at Montclair, New Jersey

The New Jersey State Teachers College at Montclair was among the first to experiment in the preparation of classroom lessons involving the use of television. Working with equipment furnished by the Allen B. DuMont Laboratories, Inc. and later with funds made available from the Allen B. DuMont Foundation, the college began in 1950 to explore and test the possibilities of television in education.

In 1952 the F.C.C. allotted UHF Channel 77 to be located at Montclair and

initial plans were made to utilize this facility. There was extensive closed circuit experimentation to college classes and to classes in the demonstration high school. On April 30, 1952 there was a full day of UHF broadcasting of planned lessons to thirteen schools in Bloomfield and Montclair, New Jersey. This was the first UHF broadcast into classrooms. It was made over the New York educational channel assigned to DuMont.

In the spring of 1954 the New Jersey State Legislature failed to authorize funds to continue educational television station WTLV which the State Department of Education had operated in New Brunswick for the previous six months. Much of the equipment and staff of this station were transferred to Montclair. The college then embarked on a detailed plan of experimentation with the aid of a grant from the Fund for the Advancement of Education of the Ford Foundation. However, after some preliminary work on the plan, it became apparent that the objectivity inherent in the college's concept of research was not compatible with the point of view of the administrators of the Fund for the Advancement of Education and the project was abandoned.

Montclair staff members prepared a two-week series of daily broadcasts of fifth-grade lessons to the public schools of Long Branch and Red Bank, New Jersey in May and June 1954. This was done over UHF station WLTV at Asbury Park assigned to Walter Reed Theatrical Enterprises.

The college first offered college credit courses in the use of television in education in 1950, when an educational television workshop was organized. The primary purpose of the workshop is to encourage students to explore the new medium and to become aware of its potentialities and inherent difficulties when used as an aid to the teacher. The workshop serves both undergraduate and graduate students in all academic areas. There are two elective courses for undergraduates. Two additional courses are available for seniors and graduate students. Each course is for two semester hours. The students gain experience in planning programs, handling television studio equipment, and producing programs for the classroom. Each student develops a program in an academic area. Thus the students (future teachers) gain an opportunity to experiment with materials and actually to produce and evaluate programs on closed circuit TV.

There has been a modest amount of work in mathematics. For example, one instructor presented a lecture-demonstration on vectors to the freshmen mathematics majors via television. Students have presented lessons on geometric solids for high school classes. A demonstration on the construction and use of surveying instruments has been televised.

Last semester undergraduate students presented two closed-circuit TV programs to a history of mathematics class. These were visualized as typical programs under the title *Mathematics, Old and New*. A twenty-minute program, *Computing Machines, Old and New*, was designed to trace the development of computers, discussing in detail the sand table, abacus and desk calculator. Production difficulties made it necessary for the students to design special models of a sand table (done as a plaster mould) and an abacus. The other

student program, *Inventions of Napier*, included a discussion with models of Napier's rods and a development of his spherical triangle formulas. Evaluations of the programs indicated that both shows brought close-up views of the models to the class that could not have been attained in a normal classroom situation.

There have been many audio-visual activities associated with Montclair. Among the films are: (1) *Not By Chance*, a National Educational Association film on teacher training which has been shown throughout the country; (2) *Together for Children*, filmed for the New Jersey Educational Association showing the history of education in New Jersey; and (3) *Resources Limited*, dealing with conservation in New Jersey. A series of four films on *Patterns of Good Teaching* is now in progress supported by a grant from the Danforth Foundation of St. Louis, Missouri. This project is concerned with the filming of outstanding teachers in action.

Exploration with films and TV is continually going on at Montclair even though at a modest rate. There is a trained staff. There are opportunities for students to gain considerable experience. Should expanded closed-circuit or UHF transmission facilities become available in this area, these students would be ready to initiate and make effective use of TV programs.

The history of TV at Montclair seems to reflect the problems of many efforts at educational TV—an impressive start, inadequate funds, very modest achievements in specialized subject areas such as mathematics, continued hard work and achievements by a small group of capable local staff members.

BASIC HIGH SCHOOL MATHEMATICS

JOSEPH SPEAR, Northeastern University

An engineering corporation located near Boston, Massachusetts, desired to have certain of its employees from the machine shops and the drafting and engineering departments review some basic mathematics in order to prepare themselves for further study in mathematics and to upgrade themselves in their work. As a result, Northeastern University is now giving a weekly half-hour program reviewing basic high school mathematics over WGBH-TV, the educational broadcasting station in Cambridge, Massachusetts.

This is the first time that a regular course in mathematics has been presented live over TV in this area. It is entirely an experiment and, to our great satisfaction, we are finding tremendous interest throughout the listening audience. This is our twelfth week* and already we have received comments and questions from about 1200 persons. These include teachers and principals of secondary schools, high schools, and colleges; lawyers, doctors, ministers; in-

* This article was written in January, 1958.

dustrial firms; engineers, soldiers, college graduates; and many mothers and school students.

We have found that the half hour is too short a period to discuss the topics without hurrying a bit. The course was originally intended as a review for those who needed it in their work, but we have found that many others, out of school for many years, are enjoying it, listening to it regularly, and interested in refreshing their memories. Much attention to the broadcasts is paid by students presently in high school and by their parents. Many other adults, perhaps for the first time, are recognizing that mathematics is a live and interesting subject. In many cases, fears traditionally associated with mathematics are being dispelled.

Since this present series seems so satisfactory, we expect to give, early in the summer of 1958, a regular high school mathematics review course for teachers who are not teaching mathematics, but who have at some time studied it, and who desire to refresh their knowledge and may thus be able to fill the vacancies now open for mathematics teachers. These teachers will enroll in the regular course and follow the lectures over TV one hour a night, four nights a week. They will do assigned work and attend classes at the University on Saturday mornings for discussion, classroom work, and examinations. The course will run for six weeks and will carry some credit towards the Master's degree in Education.

TEACHING CALCULUS BY CLOSED-CIRCUIT TELEVISION*

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Among the many questions concerning instructional television that have been raised in the past two or three years are several that deal with the feasibility of televised mathematics instruction. Related to this, the progress of Washington University's program of televised mathematics instruction is familiar to many teachers of mathematics. Some may also know of the calculus course currently being offered in the Washington, D. C., area under the auspices of the National Academy of Sciences. Both of these programs are of considerable interest, but neither has yet provided information to permit an empirical test of the effectiveness of such instruction.

We suggest that the following report is of interest for two main reasons. First, it supplies information gathered under conditions of adequate control

* This article contains much of the information to be found in the following mimeographed report: W. F. Seibert, A Brief Report and Evaluation of Closed-Circuit Television Instruction in the First Semester Calculus Course. Purdue University, Lafayette, Indiana, July, 1957. Copies of this somewhat more detailed report are available on request.

and helps to answer questions concerning the effectiveness of televised mathematics instruction. Second, both the methods of the present study and its findings are similar in broad outline to the majority of other studies of televised instructional effectiveness. Thus, the report can serve as an introduction to this area of educational research. Those with time and interest to devote to further examination of TV research studies can secure report copies from such widely distributed universities as Pennsylvania State [1], [9]; New York [8]; Toronto [13]; Iowa [12]; Miami, Ohio [5], [6]; Houston [3]; and Purdue [10], [11]. An inventory of many instructional TV research studies is also available [4].

The following note of caution may not be necessary but it reminds each reader that infallible criteria for measuring educational outcomes are so rare as to be practically nonexistent. In the present study, criteria have been used for which some validity can be claimed, but other, equally valid criteria might have been used and it is possible that their use would have had a noticeable effect upon the findings of the study. For example, an argument can be made for using measurements of long-term student retention of knowledge and skill rather than learning immediately measured, as in the present study, and something less than a perfect correlation can be assumed to exist between measurements of these two types. These ideas should be borne in mind as one examines the material to follow.

The Study

Purposes. The main problem to be studied is the comparative effectiveness of televised and conventional first-semester calculus instruction. In addition, results will be examined to determine if televised instruction has proved either more or less appropriate for students whose past academic records were superior, average, and inferior. Attitudes will be measured to determine student feelings toward TV instruction and the ways in which these feelings are influenced by experience in a televised course.

Instructional Conditions. At Purdue, as at many other institutions, first-semester calculus is a four credit-hour course normally taken during the student's second year in college. Students in the course are tested periodically (in this case, six tests were given) and homework is assigned regularly. Homework assignments are not carefully graded because of the time demands upon the instructor which this would entail but records are kept of the number of assignments that each student submits. Students are expected to attend four 50-minute class meetings per week.

Calculus is normally taught by means of small-class teaching methods, but information in the *Encyclopedia of Educational Research* [7], pp. 212–215) and experience in the use of the large-class (predominantly lecture) methods suggest strongly that they are as effective as the more customary small-class methods. In this study, large-class methods were employed three periods per week for instruction of the approximately 90 students in the control group. The fourth period of instruction was devoted to small-class meetings of about 23 students

each, under the direction of a graduate assistant. Use of the large-class method of control-group instruction made it possible to achieve experimental controls which would not have been possible otherwise. All students received the same examinations, administered simultaneously in experimental and control sections. The two principal teachers spent equal time teaching conventionally (large-class) and by television.

The four experimental sections each numbered approximately 25 students who met three times per week for instruction by television. A fourth meeting for these sections utilized the same instructional procedures and teaching personnel as were employed for the fourth class meeting of the control group. Each television student was assigned to one of four viewing classrooms. Each room contained an ordinary 24-inch television receiver mounted on a moveable pedestal with the center of the picture tube approximately 5½ feet above the floor. Receivers were connected with the campus television studio by coaxial cable. No proctors or supervisors were regularly assigned to oversee the television classes but faculty visitors were occasionally present.

Instructors. Two teachers of professorial rank in the Department of Mathematics were responsible for the large-class and televised instruction. Both teachers have the Ph.D. in mathematics and many years' experience teaching the course under consideration. One was responsible for control-group instruction during the first half (eight weeks) of the semester while the other taught the experimental group by television. At mid-semester, the instructors exchanged teaching responsibilities.

Students. The group of experimentally-taught students numbered 103 and the control group numbered 91. Of these two groups, all but three were male, 90% were first semester sophomores, 69% were in the 18–20 year old age range, and 90% were engineering students. Eighty-seven TV students and 73 control students had taken the *Purdue Mathematics Training Test* prior to enrolling for the calculus course. The respective average raw scores of these two groups were 48.7 and 49.2. Cumulative grade-point indexes (used as a basis for matching or equating students from the two groups) were available for 91 TV and 77 control students. The respective averages were 4.37 and 4.54 (4 = C, 5 = B, *etc.*). These indexes were based on work that students had completed by June, 1956, and for the majority of students represent their average performance in courses taken as freshmen.

Criteria. Two types of criteria were used in the analyses of this study. Student achievement was measured by six tests administered during the semester. Student reactions or attitudes were measured by a ten-item questionnaire administered to most participating students both before and after the semester's instruction.

The six tests were written by the course instructors and graded by them and their assistants. The grading procedure was one which should minimize irrele-

vant or inconsistent marking methods. In advance of grading, standards were agreed upon by the graders and each grader accepted responsibility for one portion of the test to be marked. They then scored an assigned portion of each test paper, without regard to the student group from which it had come.

Achievement Comparisons. Sixty-one pairs of matched students served as subjects for the comparisons of achievement of students in the experimental and control sections. Each of the 61 students from the experimental group was matched with a counterpart from the control group, using the student's cumulative grade-point index as the matching variable. In the TV group, this grade-point index correlated .65 with students' summed scores on the six calculus tests. The correlation was .55 in the control group. None of the achievement-test score data was available or used during this matching operation.

At Purdue, students' cumulative grade-point indexes carry two places to the right of the decimal, *e.g.*, 3.47. Twenty-three perfect matches were obtained, *i.e.*, grade point differences of 0.00 existed between the paired students. Eleven differences were 0.01, ten were 0.02. No matched pairs exhibited differences greater than 0.07 and there was only one such difference. The grade-point indexes of these matched students ranged from 5.40 (almost midway between an A and a B) down to 3.27 (just above a D). Average values describing characteristics of the matched groups are presented below.

	Matched Groups	
	Experimental	Control
Age	20.6	21.3
Cum. gr. pt. index	4.14	4.15
Purdue Math. Trng. Test*	44.9	42.8

The matched groups may be accepted as fairly representative of the original groups from which they came except that matched groups are composed only of students who had attempted all six calculus tests and had established a cumulative grade-point index at Purdue. The original experimental group missed 6%, the control group 4% of the calculus tests.

Scores on each of the six calculus tests and the summed scores from all six tests have been compared statistically by means of the *t*-test between correlated means ([2], pp. 278–282). For results of these comparisons, see Table 1.

Student Reactions Analysis. As mentioned earlier, most students in experimental and control sections answered a ten-item questionnaire both before and after the semester's instruction. Each of the ten items provided students with four response alternatives: Agree; Undecided, probably agree; Undecided,

* The correlation between the *Purdue Mathematics Training Test* and students' summed scores on the six calculus tests was found to be .30 in the TV group and .28 in the control group. These coefficients may be compared with the .65 and .55 which were found to exist between students' cumulative grade-point indexes and the same summed scores.

probably disagree; and Disagree. For analysis purposes, these responses were dichotomized with the "agrees" and "probably agrees" separated from the "disagrees" and "probably disagrees". Also, the analysis of student reactions treats all available pretest and posttest student responses, rather than responses from matched students only. A summary of student responses to the questionnaire is presented in Table 2.

Results. Table 1 presents matched-group test-score averages for each of the six calculus tests and for the summed score from all six of these tests. It also presents the *t*-ratio for the observed differences between comparable averages, standard deviations of the score distributions, and test-score averages for students whose past academic records were superior, average, and inferior. The superior subgroups are composed of students whose cumulative grade-point indexes range from 4.33 to 5.40 ($N=20$, each group), the average subgroups are those with indexes from 3.95 to 4.31 ($N=21$, each group), and the inferior subgroups are those students who had indexes from 3.27 to 3.93 ($N=20$, each group). No statistical tests were applied in connection with these subgroup performance comparisons.

TABLE 1

COMPARISONS OF MEAN TEST SCORES, TV AND CONTROL GROUPS, AND RELATED INFORMATION

Student Achievement Variable	Mean Scores and Standard Deviations		"t"	Remarks	Sub group Comparisons		
	TV Group	Control Group			Past Academic Record	Mean Scores TV	Control
Test 1	56.49	57.20	0.253	Not statistically significant	High	64.7	66.3
					Medium	59.0	53.4
	18.19	17.70			Low	45.7	52.1
Test 2	52.46	65.54	5.032	Significant beyond the .01 level of confidence	High	64.1	77.0
					Medium	48.6	63.2
	16.12	16.69			Low	44.9	56.6
Test 3	51.92	48.07	1.350	Not statistically significant	High	59.4	59.5
					Medium	54.0	44.7
	19.34	15.95			Low	42.4	40.2
Test 4	62.30	68.02	2.115	Significant beyond the .05 level of confidence	High	70.2	70.5
					Medium	63.3	69.4
	17.13	11.94			Low	53.4	64.2
Test 5	58.25	55.02	0.977	Not statistically significant	High	63.7	63.7
					Medium	61.6	55.4
	20.63	22.64			Low	49.2	45.9
Test 6	59.10	58.77	0.129	Not statistically significant	High	63.0	66.8
					Medium	58.1	57.1
	13.86	15.37			Low	56.2	52.5
Sum of 6 tests	340.51	352.61	1.079	Not statistically significant	High	385.0	403.7
					Medium	344.5	343.2
	74.52	75.95			Low	291.8	311.4

Referring to Table 1, it will be noticed that, of the seven tests of significance, two provide information sufficient for rejecting the null hypothesis. Both of these statistically significant differences favor the control group. Of the five remaining nonsignificant differences, two relate to the occurrence of higher scores in the control group and three relate to higher scores in the experimental group. It should be noted that the average summed calculus test score of the control group is 352.61, while the corresponding score for the experimental group is 340.51. The difference is not statistically significant.

An inspection of the subgroup averages presented in the last two columns of Table 1 reveals nothing to suggest that these averages require separate interpretation. If there is any consistent pattern in them, it would seem to be that the test performance of comparable subgroups follows the same pattern as that of the larger groups. Among the 21 pairs of comparable subgroup averages, eight show at least slight differences favoring the experimental group, one is a "tie", and twelve show at least slight differences favoring the control group. These findings are discussed later.

Procedures employed in the analysis of student questionnaire responses were described earlier. Findings pertaining to the questionnaire responses are summarized in Table 2.

In response to item 1 of the questionnaire, from two thirds to three quarters of the students indicated at least some feeling that televised classroom lectures did not instruct as well as conventional lectures. Differences between pretests and posttests and between TV and control students are generally small. On question 2 pretest, over half of the TV students felt that televised demonstrations instructed as well as conventional demonstrations; however, on posttest slightly less than half continued to feel this way. On question three, about two thirds of the TV student pretest responses indicated at least a tendency to believe that televised instruction is not as interesting as conventional instruction. More than three quarters of the group made this response on posttest. The responses of the control group to question 3 were very similar to those of the TV group.

On question 4, more than two thirds of the TV group pretest responses favored the view that television-taught students are more comfortable than conventionally-taught students. This percentage climbed to 83 on posttest. In the control group, only slightly more than half of the students gave similar responses. Question 5 can be summed up quickly. An overwhelming majority of students feel that the absence of discussion opportunities in TV classes is a serious limitation of the TV method used. Question 6 provides some interesting responses. On this item, about 40 per cent of the experimental group feel that TV students can take better notes than conventionally-taught students. In the control group, less than 20 per cent shared this view. Question 7 is another one which can be rapidly summarized. Most students agree that TV students do not get to know their instructor as well as students who are conventionally taught.

Item 8 produces some interesting responses. On both pretest and posttest, the experimental group was about evenly divided on the question of whether or not TV students are inclined to do more homework than conventionally-taught students but only a quarter to a third of the control group made similar responses. Item 9 again shows an interesting difference between responses of the experimental and control groups. Almost two thirds of the TV students began

TABLE 2

ATTITUDE ITEMS AND RESPONSE PERCENTAGES FOR EXPERIMENTAL AND CONTROL GROUPS

ITEM	RESPONSE	Experimental Group		Control Group	
		Pretest %	Posttest %	Pretest %	Posttest %
1. A televised classroom lecture does not instruct as well as the "conventional" classroom lecture.	Agree or prob. agr.	64	76	70	78
	Disagree or pr. dis.	36	24	30	22
2. A televised classroom demonstration (of materials, procedures, *etc.*) instructs as well as the "conventional" classroom demonstration.	Agree or prob. agr.	61	45	55	59
	Disagree or pr. dis.	39	55	45	40
3. Instruction received over classroom television does not capture and hold student interest as well as instruction received in the "conventional" class.	Agree or prob. agr.	65	78	68	82
	Disagree or pr. dis.	34	22	32	18
4. Students are more comfortable, more at ease, when receiving televised instruction than when attending a "conventional" class.	Agree or prob. agr.	68	83	58	52
	Disagree or pr. dis.	32	17	42	48
5. The inability of students to discuss questions with the instructor during class is a serious limitation of TV instruction.	Agree or prob. agr.	97	98	90	93
	Disagree or pr. dis.	3	2	10	7
6. Students can take a better set of notes when attending TV classes than when attending a "conventional" class.	Agree or prob. agr.	36	41	19	18
	Disagree or pr. dis.	64	59	80	82
7. Students who attend a TV class do not get to know their instructor as well as they might if they attended a "conventional" class.	Agree or prob. agr.	94	96	84	91
	Disagree or pr. dis.	6	4	15	9
8. Students who attend a TV class will be inclined to do more homework and extra reading than those who attend a "conventional" class.	Agree or prob. agr.	47	50	36	24
	Disagree or pr. dis.	53	50	64	76
9. Classroom discipline is a bigger problem in the TV classroom than in the "conventional" class.	Agree or prob. agr.	64	54	62	74
	Disagree or pr. dis.	36	46	38	26
10. Television may be all right for entertainment, but it does not belong in the classroom.	Agree or prob. agr.	37	61	49	61
	Disagree or pr. dis.	63	39	51	39

the semester with some feeling that TV classroom discipline would be a larger problem than discipline in conventional classes. This was scaled down somewhat to 54 per cent by the end of the semester. However, in the control group, 62 per cent began the semester with some doubts about discipline in TV classes and this rose to 74 per cent on the control group posttest.

Posttest responses to question 10 show that 39 per cent of the students in each group tended to accept television as a device for classroom use. In the TV group, there was a significant decline in their acceptance of this idea since, at the beginning of the semester, 63 per cent had indicated a similar view.

Discussion. In considering the evidence of student achievement in the experimental and control sections, it is necessary to think of practical as well as statistical significance of the observed differences in group performance. It is also necessary to consider students' total performance as well as their performance on each of the several achievement tests. With these things in mind, it would seem that there is little reason for believing that the televised instruction was not as effective as the instruction received by the control groups. This remark rests heavily upon the fact that, in terms of total (summed) performance on the six calculus tests, the difference between the averages of the two student groups was small and nonsignificant. Differences this great and greater can undoubtedly be found between the performances of students taught by different but still qualified instructors. Even the effectiveness of a single qualified instructor may fluctuate to produce differences of comparable magnitude.

There is little evidence to suggest that the television method operates either in favor of or against students whose past academic records were above average, average, and below average. In Table 1, it can be seen that the interaction of students' past performance and methods of instruction is, at best, inconsistent and probably nonexistent.

Student attitudes toward televised instruction are not easily summarized. The following things, however, seem to be true. Students in the television sections did alter some of their feelings toward televised instruction during the course of the semester and, predominantly, these changes resulted in a view which was less favorable toward TV than the one initially held. Some exceptions to this exist in the slight TV group response changes on items 6 and 9 of the questionnaire. It also seems true that the negative reactions of the TV group amount to considerably less than complete rejection of such instruction. At the end of the semester, about four tenths of the students in both groups tended to feel that television had some place in the classroom and about the same percentage of TV students felt that they could take better notes than students in conventional classes. Half of them believed that TV classroom discipline was not a larger problem than in conventional classes.

Posttest responses of the control group present some interesting contrasts with those of the TV group on four of the ten questionnaire items. The control students did not perceive the TV classroom as being nearly as "comfortable"

as did the television students. Neither did any significant portion of the control group students feel that TV classes permitted the taking of notes superior to those from a conventional class. Only a fourth of the control group students felt that TV students would do more homework than normal, but half of the TV group felt this was true. About three quarters of the control group students had some doubts about the discipline situation in TV classes. This compares with 54 per cent making the same responses in the TV group.

Some of the contrasts noted above may be within the limits of sampling error or might be traced back to initial biases in the groups; however, it would seem that there are some apprehensions held by students with no experience in a TV class situation which are not shared by students with such experience.

Conclusions. Information gathered and examined in the course of this study would seem to support the following conclusions:

1. Student achievement in calculus, as measured by six classroom tests, is very nearly the same for conventionally- and TV-taught students. Although two statistically significant differences were found when comparing performance of the experimental and control groups, none was found when comparing total semester's performance of these groups.

2. There are no consistent indications that televised instruction operated either in favor of or against students whose past academic records were above average, average, or below average.

3. Television students' reactions to instruction by means of TV tend to be somewhat less favorable following exposure to this form of teaching than they were initially.

4. The final reactions of television-taught students, though negative, still give some indications of noticeable acceptance.

5. Although not directly supported by the evidence, there is reason to believe that the level of achievement of television-taught students can be gradually raised as experience with and understanding of this form of instruction are increased. Such belief stems primarily from the fact that all personnel in the present study (teachers, students, and TV production) had, at best, limited experience in televised classroom instruction.

6. An initial apprehension over the limited amount of blackboard information which could be transmitted via television was diminished. Apparently the television picture can include a sufficient quantity of such information for transmission to the class, or else such visual information plays an insignificant part in the determination of student achievement.

Addendum. During the first semester of 1957–58 a second experiment on the teaching of college mathematics over closed-circuit television was carried out at

Purdue. It differed from the one described above in several ways. The subject matter was roughly that of *Fundamentals of College Mathematics* (Johnson, McCoy and O'Neill), without the calculus. About 160 students were divided in a random manner into two groups. The students in one of these (called the test group in that which follows) were divided into four smaller groups alphabetically by last names. These groups met five times a week in separate rooms. They were taught for about 30 minutes each day over closed-circuit TV. For the remaining 20 minutes of each period they were taught by graduate assistants, who concentrated on answering questions on the lectures or on assigned problems.

The second group of eighty students (the control group) met five times a week in a large lecture room. It was originally planned that they would receive each day a lecture of about 30 minutes in length, after which they would be helped by two graduate assistants. However, the instructor in charge during the first half of the term felt that these last twenty minutes were largely wasted and, after about the first two weeks, instruction was by informal lectures, during which a special effort was made to obtain student participation.

A detailed analysis of the results has not yet been made but it appears that they were very similar to those in the first experiment. It is interesting to note, however, that there were some apparently significant differences in accomplishment between different TV groups, where the breakdown was alphabetical.

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TEACHING FRESHMAN MATHEMATICS BY TELEVISION

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1. Introduction. In the Fall Semester, 1956–57, Washington University initiated a program of teaching its freshman course, Mathematics 115, College Algebra and Trigonometry, by televised lectures and accompanying help sections. Mathematics 115 is a five-hour course normally taken by science majors and students in engineering and architecture in the first half of the freshman year. Incoming students who can pass a qualifying examination on the material of Mathematics 115 may go directly to a course in analytic geometry and calculus. However, at the present time very few of our students are able to pass this qualifying examination. The prerequisite for Mathematics 115 is three or more years of high school mathematics, including at least one and one-half years of algebra.

2. Plan of operation. The material of the course was divided into 58 lessons with a text and problem assignment for each lesson. There was a 45-minute lecture on each lesson. (The lectures were kinescoped in advance during the preceding summer by H. M. MacNeille, R. R. Middlemiss, and myself.) The students had five opportunities a day to hear the lecture: it was televised to the community at 3 and 7 P.M. by the St. Louis educational television station, and was given over closed circuit in rooms on campus at 9, 10, and 12 in the morning. Students could view the 3 and 7 P.M. lectures either in rooms on campus that had been equipped with television sets or in any other place a set was available to them.

Three two-hour examinations and a three-hour final examination were held in the evening on campus. In addition to the kinescoped lectures on the 58 lessons, the day before each examination there was a live review lecture and a few days after each examination there was a live discussion lecture on the examination (both given via television).

Help sections staffed with assistants were maintained daily from 9 A.M. to 3 P.M. and from 8 to 10 in the evening for students who had questions on the lesson and needed personal help. Students were given complete freedom in regard to hearing the lectures and attending the help sections.

At registration each student in the course was given a detailed set of instructions and a lesson schedule containing the text and problem assignments. The scheme for computing mid-term and semester grades was set out explicitly in the instructions.

3. Help sections. The help sections formed an integral part of the program. They were conducted on an informal basis. There were no lectures in them; help was on a person-to-person basis. Some students would come only for a few minutes to ask one or two particular questions; others stayed for three or four hours working on the assignment and asking for help as they needed it.

There seemed to be little passive copying of problems. Attendance fluctuated with the difficulty of the assignment. The help sections proved especially valuable for weak students and for students who had been out of school for a number of years before entering the course.

The assistants in the help sections were, for the most part, junior and senior students who had had at least 18 hours of college mathematics with A's and who were taking more advanced mathematics courses at the time. In addition, there were several faculty assistants, and Professor Middlesmiss and I worked in the help sections as much as we could. The number of assistants needed per hour varied from two in the evening hours to as many as eight or nine during the most crowded daytime hours. One valuable aspect of the program was the training given the undergraduate assistants.

4. Homework and grades. Semester grades were computed on the basis of 1000 points. Each two-hour examination counted 175 points, the three-hour final counted 275 points, and homework counted 200 points. Counting homework this much forced a student to do it regularly, yet no student could pass on homework alone. As an added incentive to do the homework, we deducted five points for each homework paper a student failed to hand in. (Provisions were made for late homework papers.)

The careful handling of the homework papers and grades for the course involved an enormous amount of work for those of us who administered the course. However, we felt this to be essential with the type of student we had. Though it may be deplored, most of our students work for grades rather than for any love of mathematics. Hence we set up a system in which the student saw clearly the relationship between his grade and his performance.

5. Use by other colleges. The lectures were also followed by groups at two other colleges in the local area: Harris Teachers' College and Lindenwood College. The students in these groups took our examinations but had their own help sections. In addition, the lectures were used by a group of specially selected high school students at one local high school. These latter students averaged higher on the examinations than did our own students.

Qualified persons in the St. Louis area could take the course for credit through the evening school division of Washington University. We had 37 people registered for it in this manner. In addition, the course was followed informally by many high school students and teachers, as well as by other adults in the community.

6. Evaluation of the program gradewise. We deliberately raised the standards in the course by using more difficult, searching examinations and by raising the average required for a given letter grade. It was felt that the level of performance which was given a D this year would have been given a C last year. Despite this, the grades rose sharply, as illustrated below.

Grade	Televised Course, Fall 1956–57		Traditional Course, Fall, 1955–56	
	Number	Per Cent	Number	Per Cent
A	95	20.0	47	12.2
B	122	25.7	69	17.8
C	102	21.5	99	25.6
D	56	11.8	17	4.4
Incomplete	52	10.9	84	21.7
F	22	4.6	44	11.4
Nx	0	0.0	4	1.0
Withdrawn	26	5.5	23	5.9
TOTAL	475	100.0	387	100.0

7. Evaluation in other ways. The adoption of this new method of instruction was in large measure prompted by the necessity for finding a solution to the problems of inadequate classroom space and the impossibility of obtaining enough competent instructors to handle the rising enrollment in small sections. Besides meeting these problems, the program had other things to recommend it. One unexpected dividend was the change in the attitudes of many of the students toward mathematics; students in the new plan showed far more interest and enthusiasm for the material of the course and worked much harder than had former students in the course. The new method placed the responsibility for learning the material squarely on the student; it seemed to shift the emphasis from sitting in class to doing the homework and learning the material.

The new program served the heterogeneous freshman group which we have better than did the old one. The bright students were not forced to listen to repeated explanations of what they already knew; the slower students could get all the help they needed. The latter could, and frequently did, hear the lecture more than once.

One of the big gains we hope to achieve from the program is an improvement in the mathematical backgrounds of our incoming students, a large number of whom are from the surrounding area. We hope that in the future many of the local high schools will arrange to use our televised lectures in a course for their interested better students so that these students will not need to take the course in college, but can start immediately with analytic geometry and calculus. We feel that our lectures are also of value in informing the high school mathematics teachers as to the kind of mathematics courses for which their students should be prepared.

Because of the rather elaborate machinery necessary to operate a course in this manner and the large initial expenses, such a program is probably not suitable for courses with a small enrollment. On the other hand, the cost of operating the program—including the number of faculty members needed for it, classroom space, and other costs—increases only slightly with a substantial

increase in the number of students in the course, a factor which would recommend it for large basic courses.

Two pertinent questions are: (a) just what role does the use of television play in such a program, and (b) to what extent could the program be used by institutions that do not have access to the facilities of a television station. We feel that the closed circuit lectures could well be replaced by large (non-television) lectures to groups of 100–200, provided suitable lecture halls are available. We were unhappy with the closed circuit, partly because, despite continued efforts of technicians, it never worked very well. In addition, the noise it created was unpleasant for other classes in the building. A lecture over the closed circuit viewed in a classroom is probably not as effective as an actual classroom lecture. On the other hand, a televised lecture watched by an individual at home or by a small group in a fraternity house or dormitory is perhaps superior in many respects to an average classroom lecture. In practice, most of our students chose to watch the lecture in the evening at home or in their living group.

We felt that the success of the program was due for the most part not to the use of television but rather to the help sections and to the set-up of the course which forced the students to work.

8. Cautions. This method of instruction is no panacea for the problem of handling large enrollments with a small staff and limited classroom space. If the program is not very carefully run, we feel that it could easily fail. The task of administering such a program, particularly during its initial stages, should not be underestimated. The help sections, if they are to function efficiently, cannot be turned over completely to undergraduate assistants. At least one faculty member should be present most of the time. It should also be noted that lecturing via television requires a different technique from classroom lecturing and vastly more preparation time.

9. Continuation of the program. In the Spring Semester, 1957, the course which follows Mathematics 115, Analytic Geometry and Calculus, was given in a similar way, but without the use of a film. Two lectures a day were given on campus via the closed circuit, and one lecture was given in the evening over the educational television station. We found the live televised lectures to be far better than the kinescoped ones, both as regards clarity of the picture and the greater flexibility afforded.

The program is being continued in the present academic year (1957–58) with a few changes. The hours of the help sections have been curtailed somewhat; the examinations have been changed in form to facilitate grading. The closed-circuit lectures have been replaced by two live lectures a day given in a large auditorium. The film used last year (remade in part) is again being shown over the educational television station both in the afternoon and in the evening.

EXPERIENCE AT SEATTLE

CARL B. ALLENDOERFER, University of Washington

1. General description. Instruction in mathematics by television began at the University of Washington in January, 1956. The first course offered was Intermediate Algebra and was conducted over a period of 20 weeks through the facilities of KCTS-TV, the community educational television station for Seattle and King County. The course was officially offered through the Correspondence Department of the Division of Adult Education of the University of Washington and carried correspondence credit for five quarter-hours. On Tuesday and Wednesday evenings of each week lectures on new material were presented from 7:30 to 8:00 P.M. A help session was available at the University on Saturday mornings. Weekly homework assignments were due on Saturday morning and were returned for delivery on Monday. On Monday evening a review lesson was broadcast on which answers were given for all assigned problems and common errors in the homework were discussed. Two examinations at the University were given to registered students; grades were based on these with some credit for homework. Normal charges for correspondence courses were imposed.

A course on trigonometry was offered in a similar fashion during the winter quarter of 1957. Essentially the same pattern was followed, except that three lectures a week were given, the help session was on Thursday evening, three quarter-hours of credit were awarded, and the course ran for only ten weeks. In both courses the lectures were given by me, and the homework, help sessions, review broadcasts, and examinations were handled by Granville McCormick, a predoctoral associate. Kinescopes were made of the trigonometry lectures for possible future use.

2. Broadcast Technique. For a variety of reasons including financial stringency and lack of time for preparation, the technical aspects of the program were kept as simple as possible. Because of previous difficulties at KCTS in obtaining suitable contrast with an ordinary blackboard and chalk, a new method of presentation was devised. The lecturer was seated to the left of an easel which carried sheets of off-white newsprint. The lecturer wrote on these with a black grease pencil. One camera was trained over the lecturer's right shoulder onto these sheets, and the second camera viewed him from his left side. Excellent definition was obtained in this fashion, even in fringe areas, and the fixed position of the lecturer prevented him from wandering about the studio to the consternation of the director and camera men.

Variations in this technique were used for special purposes. Slides were prepared for the statements of word problems and for announcements; a wall chart of logarithmic and trigonometric tables proved successful in teaching the use of such tables; an oscilloscope was helpful in presenting sine curves of various periods and amplitudes; predrawn graphs of various functions were used

to improve legibility and to save time; and simple apparatus was borrowed from the Physics Department to illustrate applications of trigonometry.

There were essentially no rehearsals except for brief tests of camera angles when new devices were to be used. After a few lectures the director and the lecturer were so well coordinated that each could anticipate the actions of the other in time to prevent any serious confusion. Except for the director, the other station personnel (such as camera men and floor director) were students who were at the same moment taking a course in TV techniques. Their own enthusiasm made up for any lack in experience, and I often found myself talking to them as if they were students in my class. No classroom audience was provided and I did not find this a handicap.

3. Preparation. The outline of the course was carefully prepared in advance, and was distributed at cost to the registered students and to many others in the community as a "Viewer's Guide." After this was written, I spent about 30 minutes preparing each lecture. Most of this time was devoted to making careful notes on the solutions of problems to be presented on the air. No fumbling about for the best method of attack can be permitted (it wastes time), and numerical errors are a gross disaster, for there is no bright-eyed boy on the front row to point out a mistake. Correct timing is essential, of course; but after a little practice an experienced teacher gets a feel for 28 minutes as accurately as he does for 50 minutes.

4. Effectiveness. Comparative studies were made of the performance of the TV students as contrasted to those taught by conventional means. In addition the TV students were questioned in person and through a questionnaire on their own reactions. The conclusion we reached is that instruction on TV is just as effective as classroom instruction provided that help sessions are available and that homework is graded regularly. The students must also have access to a TV set under circumstances free from distractions. The performance of the TV students was remarkably better than that of ordinary correspondence students. In presenting theoretical developments and problems, the lecturer must be conscious of the small size of the picture and the relatively few equations that can be viewed at one time. These mean that TV is most effective in simple mathematical situations and in short problems. Algebra and trigonometry can be broken up into sufficiently small bits of this kind, but I have doubts about the effectiveness of TV in teaching analytic geometry and calculus where most situations require simultaneous viewing of a graph and fairly extensive calculations. The TV screen is just too small for this sort of thing.

5. General Conclusions. Based on this limited experience, I have reached the following conclusions—which are subject to immediate revision as my knowledge of this medium increases:

(a) The main use of TV in mathematics instruction is in Adult Education, for by this method we can reach people who are otherwise unable or unlikely

to come to the campus. Instruction in elementary subjects by this means is quite satisfactory. The effectiveness of such broadcasts upon the general public (apart from the registered students) is one of their most important features. Indeed, the public interest in our programs in Seattle was very gratifying, and they proved to be a most useful public relations device for mathematics as a subject and for the Mathematics Department of the University. It has been my experience that the public is more interested in the concrete mathematics of algebra than it is in cultural programs on the appreciation of mathematics.

(b) TV is not well suited as a replacement for normal classroom instruction of regular students. A far better method of "canning" our instructors is to make regular films to be projected on conventional movie equipment. These films, of course, can be kinescopes made in TV studios. Films can be shown at convenient hours without tying up air space, their images on the screen are much larger and clearer than on a TV set, and the equipment needed is less complicated and expensive than TV—either open or closed circuit. There is no reason why carefully prepared films presenting experienced lecturers should not be more effective than classroom teaching by young assistants. The shortage of mathematics teachers makes preparation of such films a most urgent matter for consideration by bodies such as the Mathematical Association of America.

ELEMENTARY PROBLEMS AND SOLUTIONS

EDITED BY HOWARD EVES, University of Maine

Send all communications concerning Elementary Problems and Solutions to Howard Eves, Mathematics Department, University of Maine, Orono, Maine. This department welcomes problems believed to be new, and demanding no tools beyond those ordinarily furnished in the first two years of college mathematics. To facilitate their consideration, solutions should be submitted on separate, signed sheets, within three months after publication of problems.

PROBLEMS FOR SOLUTION

E 1321. *Proposed by C. W. Bostwick, Riverdale, Maryland*

Prove that at a gathering of any six people, some three of them are either mutual acquaintances or complete strangers to each other.

E 1322. *Proposed by J. W. Andrushkiw, Seton Hall University*

Multiply the first p terms of the harmonic series by $(-1)^k$, the next q terms by $(-1)^{k+1}$, the next p terms by $(-1)^k$, the next q terms by $(-1)^{k+1}$, and so on, alternately, thus forming a series denoted by $H(p, q, k)$. Show that $H(p, q, k)$ is convergent if and only if $p=q$.

E 1323. *Proposed by Harry Goheen, Oregon State College*

Prove that all roots but one of the equation

$$nx^n = 1 + x + x^2 + \cdots + x^{n-1}$$

have absolute value less than 1.

E 1324. *Proposed by F. E. Clark, Rutgers University*

Suppose tic-tac-toe is turned into a game of pure chance as follows. Designate the squares by 1, $\cdots$, 9, numbering them from left to right by successive rows, starting with the top row. Place a set of nine chips, labeled 1, $\cdots$, 9, in a bag. The first player, A, draws a chip at random and enters an X in the corresponding square. The second player, B, draws at random from the remaining chips and enters an O in the corresponding square. The game ends, of course, as soon as three like entries are obtained which lie along a line (horizontally, vertically, or diagonally). Find the probability that A will win, draw, or lose the game. Also find the probability, for each drawing, that the game will end when that chip is drawn.

E 1325. *Proposed by Peter Treuenfels, Brookhaven National Laboratory, Upton, L. I., New York*

Let A, B, X denote $n \times n$ matrices. Show that a sufficient condition for the existence of at least one solution X of the matrix equation

$$X^2 - 2AX + B = 0$$

is that the eigenvalues of the $2n \times 2n$ matrix

$$R = \begin{bmatrix} A & I \\ A^2 - B & A \end{bmatrix}$$

be pairwise distinct. Here I denotes the $n \times n$ identity matrix.

SOLUTIONS

An Inequality

E 1291 [1957, 741]. *Proposed by G. B. Thomas, Jr., Massachusetts Institute of Technology*

Let a, b, c, d be nonnegative numbers such that $c+d \leqq \min(a, b)$. Prove that $ad+bc \leqq ab$.

Solution by Underwood Dudley, Carnegie Institute of Technology. We have $ad+bc \leqq (c+d) \max(a, b) \leqq \min(a, b) \max(a, b) = ab$.

Also solved by J. L. Alperin, A. G. Anderson, Winifred Asprey, Philip Bacon, D. W. Bailey, Edward Barbeau, Robert Bart, B. H. Bissinger, Guy Blondeau, Julian Braun, D. A. Breault, D. R. Brillinger, E. W. Brown, J. L. Brown, Jr., George Campagne, P. L. Chessin, W. J. Cody, Jr., Ronald Colling, A. E. Danese, J. E. Darraugh, C. W. Dodge, D. B. Dorsey, P. L. Duren, E. S. Eby, Irma Esrig, Eugene Famolari, Jr., D. P. Flemming, D. A. Freedman, M. L. Freimer, W. B. Fulks, John Gedeist, A. M. Glicksman, Michael Goldberg, L. K. Grodman, A. J. Gross, J. W. Haake, J. D. Haggard, Dunstan Hayden, Vern Hoggatt, J. R. Holdsworth, R. Holt, A. R. Hyde, Carlton Johnson, N. S. Kandalgaonkar, Geoffrey Kandall, A. F. Kaupe, Jr., M. S. Klamkin, Sandra Kohlenberg, J. D. E. Konhauser, Andrew Kraus, Lorraine Lavallee, A. I. Lieberman, Joe Lipman,

R. L. London, L. H. McFarlan, R. T. J. Mahoney, Wallace Manheimer, E. W. Marchand, D. C. B. Marsh, J. C. Mathews, William Moser, D. L. Muench, A. A. Mullin, J. B. Muskat, E. L. Nath, E. N. Nilson, C. S. Ogilvy, M. J. Pascual, D. J. Peterson, R. L. Pierce, J. L. Pietenpol, C. F. Pinzka, B. E. Rhoades, L. A. Ringenberg, Jeff Ritterman, F. E. Rote, H. D. Ruderman, Paul Schillo, D. L. Shell, Paul Slepian, G. S. Stoller, W. B. Stoval, Jr., D. R. Sudborough, Lawrence Van Cura, K. D. Ware, F. L. Wolf, David Zeitlin, and the proposer. Late solutions by C. D. Anderson, Roger Bachmann, J. Gallego-Diaz, R. H. Hou, Morton Kupperman, Jack Macki, Leo Moser, and J. B. Oenning.

The Pirate's Treasure

E 1292 [1957, 741]. *Proposed by Jose Gallego-Diaz, Vanderbilt University*

A pirate decided to bury a treasure on an island near the shore of which were two similar boulders A and B and, farther inland, three coconut trees C_1, C_2, C_3. Stationing himself at C_1, the pirate laid off C_1A_1 perpendicular and equal to C_1A and directed outwardly from the perimeter of triangle AC_1B. He similarly laid off C_1B_1 perpendicular and equal to C_1B and also directed outwardly from the perimeter of triangle AC_1B. He then located P_1, the intersection of AB_1 and BA_1. Stationing himself at C_2 and C_3, he similarly located points P_2 and P_3, and finally buried his treasure at the circumcenter of triangle $P_1P_2P_3$.

Returning to the island some years later, the pirate found that a big storm had obliterated all the coconut trees on the island. How might he find his buried treasure? (Dedicated to Howard Dachslager.)

I. *Solution by C. F. Pinzka, University of Cincinnati.* Since triangles AB_1C_1 and A_1BC_1 are congruent, angles C_1AB_1 and C_1A_1B are equal and then $\sphericalangle A_1P_1A = \sphericalangle A_1C_1A = 90°$. Thus P_1, and likewise P_2 and P_3, lie on the circle with diameter AB, and the treasure is buried midway between A and B.

II. *Solution by R. R. Seeber, Jr., IBM Corp., Poughkeepsie, N. Y.* While wondering how to proceed, the pirate watched three sea gulls engaged in a violent aerial battle. Presently all three birds fluttered to the ground, dead. Taking this as a favorable omen, the pirate proceeded with his original construction, using the locations of the three birds for replacements of the coconut trees. He found his treasure and was interested to note that it was buried at the midpoint of AB, a fact which had previously been concealed by a clump of shore-side coconut trees.

Also solved by A. G. Anderson, Robert Bart, J. L. Botsford, M. G. Boyce, Julian Braun, E. W. Brown, J. W. Clawson, C. W. Dodge, Brother Louis Francis, J. W. Gammill, J. W. Haake, A. R. Hyde, N. S. Kandalgaonkar, Geoffrey Kandall, Sister M. Kenneth Kolmer, J. D. E. Konhauser, M. A. Laframboise, Joe Lipman, Wallace Manheimer, D. C. B. Marsh, Beckham Martin, E. L. Nath, C. S. Ogilvy, D. S. Passman, J. L. Pietenpol, J. R. Sykes, Sister M. Stephanie, Ismael Torrecilla, and the proposer. Late solutions by C. D. Anderson, D. F. Atkins, Roger Bachmann, and J. B. Oenning.

A General Congruence

E 1293 [1957, 742]. *Proposed by J. B. Roberts, Reed College*

Prove that for every nonnegative integer t

$$3(1 + 6^t + 8^t) \equiv 1^t + 2^t + 3^t + \cdots + 9^t \pmod{18}.$$

Solution by Norman Miller, Queen's University. The congruence is readily verified for $t=0$ and $t=1$. Take $t>1$. The difference of the two expressions is

$$2^t + 3^t + 4^t + 5^t + 7^t + 9^t - 2 - 2\cdot 6^t - 2\cdot 8^t.$$

Since five terms are even and four terms are odd, this is divisible by 2. Three of the terms are divisible by 9. It remains to show that

$$2^t + 4^t + 5^t + 7^t - 2 - 2\cdot 8^t \equiv 0 \pmod 9. \tag{1}$$

The grouping $(7^t+2^t)+(5^t+4^t)-2(8^t+1)$ shows that this is true when t is odd. Take t even and write (1) in the form

$$7(7^{t-1} + 2^{t-1}) - 5\cdot 2^{t-1} + 5(5^{t-1} + 4^{t-1}) - 4^{t-1} - 16(8^{t-1} + 1) + 14.$$

Since $t-1$ is odd, each expression in parentheses is divisible by 9. Also

$$\begin{aligned} 5\cdot 2^{t-1} + 4^{t-1} - 14 &= 2^{2t-2} + 5\cdot 2^{t-1} - 14 \\ &= (2^{t-1} + 7)(2^{t-1} - 2) = 2(2^{t-1} + 1 + 6)(2^{t-2} - 1), \end{aligned}$$

where each factor in parentheses is divisible by 3. This completes the proof.

Also solved by Winifred Asprey, D. A. Breault, D. R. Brillinger, P. L. Chessin, T. W. Daniel, F. J. Duarte, Underwood Dudley, Louise Grinstein, J. W. Haake, R. Holt, A. R. Hyde, M. S. Klamkin, Sister M. Kenneth Kolmer, Joe Lipman, D. C. B. Marsh, C. T. Molloy, Jr., J. B. Muskat, Margaret Olmsted, C. A. Oster, F. D. Parker, W. G. Preble, Paul Schillo, R. R. Seeber, Jr., F. L. Wolf, David Zeitlin, and the proposer. Late solution by C. W. Trigg.

Limits of Two Recursive Sequences

E 1294 [1957, 742]. *Proposed by G. A. Harris, Jr., Yale University*

Having chosen two numbers a_1 and b_1 from the open interval (0, 1), define the sequences $\{a_n\}$ and $\{b_n\}$ recursively as follows:

$$a_{n+1} = a_1(1 - a_n - b_n) + a_n, \qquad b_{n+1} = b_1(1 - a_n - b_n) + b_n.$$

Prove that both sequences approach limits as $n\to\infty$, and find these limits.

Solution by B. H. Bissinger, Lebanon Valley College. By mathematical induction we can prove $a_n=a_1(1-d^n)/(1-d)$, where $d=1-a_1-b_1$. Since $|d|<1$, it follows that $\lim a_n=a_1/(a_1+b_1)$. By symmetrical considerations we have $\lim b_n=b_1/(a_1+b_1)$.

Also solved by D. S. Adorno, Philip Bacon, Edward Barbeau, Robert Bart, A. P. Boblétt, Julian Braun, D. R. Brillinger, P. L. Chessin, A. E. Danese, W. R. Derrick, Peter Duren, E. S. Eby, D. A. Freedman, Harry Goheen, Michael Goldberg, D. S. Greenstein, A. J. Gross and R. F. Potthoff (jointly), Emil Grosswald, J. W. Haake, Virginia S. Hanly and M. L. Slater (jointly), A. R. Hyde, N. S. Kandalgaonkar, A. F. Kaupe, Jr., D. A. Kearns, P. G. Kirmser, M. S. Klamkin, A. I. Lieberman and D. Solitar (jointly), Joe Lipman, D. C. B. Marsh, Norman Miller, C. T. Molloy, Jr., D. L. Muench, J. B. Muskat, E. L. Nath, C. S. Ogilvy, M. J. Pascual, D. S. Passman, Stanton Philipp, R. L. Pierce, J. L. Pietenpol, C. F. Pinzka, W. G. Preble, L. A. Ringenberg, Jeff Ritterman, Milton Rosenberg, Paul Schillo, D. L. Shell, D. L. Smith, G. S. Stoller, D. F. Templeton Jr., G. H. M. Thomas, A. L. Tritter, Chih-yi Wang, David Zeitlin, and the proposer. Late solutions by C. D. Anderson, D. F. Atkins, D. A. Breault, R. H. Hou, J. D. E. Konhauser, Morton Kupperman, Yoshio Matsuoka, Leo Moser, and O. E. Stanaitis.

A Transcendental Complex Equation with Only Real Roots

E 1295 [1957, 742]. *Proposed by M. S. Klamkin and D. J. Newman, AVCO Research and Development, Lawrence, Mass.*

Show that all the roots of $\tan z = z/(1+m^2z^2)$, where m is real, are real.

I. *Solution by J. W. Haake, Armour Research Foundation, Tucson, Arizona.* We note that if z is a root of the given equation, then so are $\bar{z}$ and $-z$. Hence, in discussing imaginary roots $z=a+ib$, $b\neq 0$, we may, for convenience, assume $a\geqq 0$, $b>0$. Now in the given equation, set $z=a+ib(b\neq 0)$, $\tan z=(\sin z)/(\cos z)$, $\sin ib=i\sinh b$, $\cos ib=\cosh b$, and then multiply each side of the resulting equation by the conjugate of the denominator of that side. We obtain

$$\frac{(\sin 2a)/2 + (i\sinh 2b)/2}{\cos^2 a\cosh^2 b + \sin^2 a\sinh^2 b} = \frac{a[1+m^2(a^2+b^2)] + ib[1-m^2(a^2+b^2)]}{[1+m^2(a^2+b^2)]^2 + 4m^4a^2b^2}.$$

Since real and imaginary parts must be equal, we obtain, by taking the ratio of real part to imaginary part on each side,

$$\frac{\sin 2a}{\sinh 2b} = \frac{a}{b}\left[\frac{1+m^2(a^2+b^2)}{1-m^2(a^2+b^2)}\right].$$

For real m, this implies the impossible inequality

$$(\sin r)/(\sinh s) \geqq r/s, \qquad r\geqq 0,\ s>0.$$

It follows that we cannot have $b\neq 0$, and z must be real.

II. *Solution by A. E. Danese, Eastman Kodak Co., Rochester, N. Y.* Let $F(z)=\tan z - z/(1+m^2z^2)$. Then the zeros of $F(z)$ are those of

$$g(z) = (1+m^2z^2)\tan z - z.$$

It is readily verified that

$$(2/z^3)\cos z\, g(z) = \int_0^1 (2m^2+1-t^2)\cos zt\, dt.$$

Application of the following theorem [Pólya and Szegö, *Aufgaben und Lehrsätz aus der Analysis*, Zweiter Band, New York, 1945, p. 69, 173] yields the result: Let $f(t)$ be twice continuously differentiable, $f(t)>0$, $f'(t)\leqq 0$, $f''(t)<0$ for $0\leqq t\leqq 1$. Then the function $\int_0^1 f(t)\cos zt\, dt$ has infinitely many, and only real, zeros.

Also solved by Robert Bart, D. C. B. Marsh, Nathan Schwid, and the proposers. Late solutions by R. H. Hou and J. D. E. Konhauser.

ADVANCED PROBLEMS AND SOLUTIONS

EDITED BY E. P. STARKE, Rutgers University

Send all communications concerning Advanced Problems and Solutions to E. P. Starke, Rutgers University, New Brunswick, New Jersey. All manuscripts should be typewritten with double spacing and margins at least one inch wide. Problems containing results believed to be new or extensions of old results are especially sought. Proposers of problems should also enclose any solutions or information that will assist the editor. In general, problems in well-known textbooks or results in readily accessible sources should not be proposed for this department.

PROBLEMS FOR SOLUTION

4776 [1958, 125]. Correction. *Proposed by D. J. Newman, AVCO Research and Development*

If $|\alpha| < 1/e$, then $\sum_{n=0}^{\infty} (z+\alpha n)^n/n!$ represents an entire function. Prove in fact that it is a simple exponential, $Ae^{\lambda z}$.

4793. *Proposed by M. H. Lietzke and C. W. Nestor, Jr., Oak Ridge National Laboratory*

In *Mathematics and the Imagination*, Kasner and Newman present the following problem. An equilateral triangle is inscribed in a circle of unit radius, another circle is inscribed in the triangle, a square in this circle, *etc.* Continue the procedure, increasing the number of sides of the regular polygon each time by one. As the number of sides of the inscribed polygon increases, the radii of the shrinking circles converge to a definite limiting value. Find this value. The proposed answer (approximately 1/12) seems to be considerably in error. What should it be?

4794. *Proposed by S. W. Golomb, California Institute of Technology*

Let V_n be binary n-space (the collection of n-vectors over the field of two elements). Consider two vectors of V_n to be in the same *class* if they differ only by a cyclic permutation of their components. Show that the number of classes is even, except when $n=2$.

4795. *Proposed by Y. L. Luke, Midwest Research Institute, Kansas City, Missouri*

Find the zeros of

$$H_n(x) = \sum_{m=0}^{n} \frac{(-2)^m(2n-m)!\Gamma(x+1)}{m!(n-m)!\Gamma(x-m+1)}.$$

4796. *Proposed by Chandler Davis, Institute for Advanced Study*

Say t_1 is *preferred* to t_2 provided (i) $0 \leqq t_1 < t_2 \leqq 1$, or (ii) $-1 \leqq t_2 < t_1 < 0$, or (iii) $-1 \leqq t_2 < 0 \leqq t_1 \leqq 1$. Determine a sequence of polynomials P_n such that, whenever t_1 is preferred to t_2, $P_n(t_2) = o(P_n(t_1))$.

4797. *Proposed by D. J. Newman, Massachusetts Institute of Technology*

Prove that all expressions like $7\sqrt{19}/4-3\sqrt{7}+8\sqrt{6}/5$ are irrational. More specifically, prove that the square roots of the square-free integers are linearly independent over the rationals.

SOLUTIONS

Fibonacci Numbers

4747 [1957, 437]. *Proposed by B. J. Boyer, Lafayette, Ind.*

Write $S_n=\sum_{k=0}^{n} B_k^2$,* where the B_k are Fibonacci numbers, $B_0=1$, $B_1=1$, $B_{n+2}=B_{n+1}+B_n$. Find the value of $\sum_{n=1}^{\infty}(-1)^n/S_n$.

Solution by D. C. B. Marsh, Colorado School of Mines. We have

$$S_n = B_n \cdot B_{n+1}, \tag{1}$$

$$\sum_{n=0}^{m} (-1)^n/S_n = B_m/B_{m+1}, \tag{2}$$

$$\lim_{m\to\infty} B_m/B_{m+1} = \tfrac{1}{2}(\sqrt{5}-1), \tag{3}$$

which is the desired result. (1) and (3) are well known (cf. this MONTHLY, vol. 60, 1953, p. 680 ff.) and (2) is easily demonstrated by induction.

Also solved by W. J. Blundon, Robert Breusch, P. G. Engstrom, Calvin Forman, J. Ginsburg, L. D. Goldberg, S. H. Greene, Emil Grosswald, A. R. Hyde, M. S. Klamkin, T. F. Mulcrone, J. K. Muskat, J. L. Pietenpol, R. C. Read, D. A. Robinson, D. L. Scheffler and R. A. Sebastian, Chih-yi Wang, E. J. Williams, L. K. Williams, and the proposer. Late solution by Yoshio Matsuoka.

Editorial Note. Robinson calls attention to the fact that the problem was proposed by Lucas (1) and solved by Mangon (2). This result contained an error which was noted in the same journal (3). Dickson (4) states Mangon's incorrect result but fails to take note of the later correction.

1. *Nouvelle Correspondance Mathematique*, vol. 6, 1880, p. 418.
2. *Ibid.*, p. 420.
3. *Ibid.*, p. 480.
4. Dickson, L. E., *History of the Theory of Numbers*, vol. 1, 1919, p. 402.

Rational and Irrational Numbers

4748 [1957, 509]. *Proposed by Alexander Oppenheim, University of Malaya*

Suppose that x and y are defined by the two series

$$x = \frac{1}{a_1} + \frac{1}{a_1^2}\frac{1}{a_2} + \frac{1}{a_1^2 a_2^2}\frac{1}{a_3} + \cdots,$$

$$y = \frac{1}{a_1} + \frac{1}{a_1^3}\frac{1}{a_2} + \frac{1}{a_1^3 a_2^3}\frac{1}{a_3} + \cdots,$$

* The upper limit of the summation was originally printed ∞ in error.

where the a_i are integers such that $a_i \geqq 2 (i = 1, 2, \cdots)$. Prove that x and y are both rational or irrational.

Solution by Rimhak Ree, University of British Columbia. First we prove a lemma: *Let z be a positive rational number in* $(0, 1)$, *and let* $e \geqq 2$. *If the inequality* $0 < (za - 1)a^{e-1} < 1$ *has a positive integral solution* a, *then the solution is unique.* If b is another integral solution, (we may assume $a < b$) and $z = m/n$ with integers m, $n > 0$, then we will have

$$\frac{n}{a} < m \leqq m(b - a) < \frac{n}{a^{e-1}}$$

which is a contradiction. Thus the lemma is proved.

Let e be an integer $\geqq 2$, and set

$$z_i = a_i^{-1} + (a_i^{\cdot} a_{i+1})^{-1} + (a_i^{\cdot} a_{i+1}^{\cdot} a_{i+2})^{-1} + \cdots .$$

Then $0 < z_i < 1$, and $(z_i a_i - 1)a_i^{e-1} = z_{i+1}$. Suppose z_1 is a rational number of the form m_1/n with integers m_1, $n > 0$. Then z_i is a rational number of the form m_i/n with $0 < m_i < n$. Since there are at most $n - 1$ distinct values for m_i, there are positive integers j, k such that $z_j = z_{j+k}$. Then by the lemma, $a_j = a_{j+k}$ and hence $z_t = z_{t+k}$, $a_t = a_{t+k}$ for all $t \geqq j$. It is easily seen that if there are positive integers j, k such that $a_t = a_{t+k}$ for all $t \geqq j$ then z_1 is rational. Thus z_1 is rational if and only if there exist integers j, $k > 0$ such that $a_t = a_{t+k}$ for all $t \geqq j$. Now, $x = z_1$ if $e = 2$, and $y = z_1$ if $e = 3$. Hence x and y are both rational or both irrational.

Also solved by Andrew Kraus, Bladovest Sendov, and the proposer.

Editorial Note. Suppose that

$$w = \frac{1}{a_1} + \frac{1}{a_1 b_1}\frac{1}{a_2} + \frac{1}{a_1 b_1 a_2 b_2}\frac{1}{a_3} + \cdots ,$$

where a_i, b_i are infinite sequences of integers which satisfy the conditions $b_i \geqq a_i - 1 \geqq 1$, $(i = 1, 2, \cdots)$ and $b_i = b_j$ whenever $a_i = a_j$. Then, as shown by the proposer, an argument similar to the above proves that w is rational if and only if the sequence (a_i) (and therefore also the sequence (b_i)) is ultimately periodic.

Unbounded Fourier Coefficients

4749 [1957, 509]. *Proposed by Joseph Lehner, Michigan State University*

Is it true that for every set E of positive measure contained in the interval $(0, 1)$, the numbers C_n have the property $|nC_n| < M$, $n = 1, 2, \cdots$, where

$$C_n = \int_E e^{2\pi i n x} dx,$$

and M is a constant depending only on E?

Solution by Fritz Herzog, Michigan State University. We shall construct an open set E in $(0, 1)$ for which $\{nC_n\}$ is unbounded.

For positive integral p and for $q=1, 2, \cdots, 2p$, let E_{pq} denote the interval $(q-1/2)/(2p)!<x<q/(2p)!$. It is easily verified that no two of these intervals are overlapping. Let

$$E_p = \bigcup_{q=1}^{2p} E_{pq}, \qquad E = \bigcup_{p=1}^{\infty} E_p.$$

We shall show that, for the set E defined above, $nC_n \to \infty$ as $n \to \infty$ through the values $(2m)!$

For fixed m, let the integral

$$\int_X \exp\left[2\pi i(2m)!x\right]dx$$

be denoted by $I(X)$. Then

$$(1) \qquad I(E_{pq}) = \frac{\exp\left[2\pi iq(2m)!/(2p)!\right] - \exp\left[2\pi i(q-1/2)(2m)!/(2p)!\right]}{2\pi i(2m)!}.$$

The numerator on the right side of (1) vanishes when $p<m$, and it equals 2 when $p=m$. Hence

$$(2) \qquad C_{(2m)!} = I(E) = \frac{2m}{\pi i(2m)!} + I(E_{m+1} + E_{m+2} + \cdots).$$

To estimate $I(E_{m+1}+E_{m+2}+\cdots)$, we note that $E_{m+1}+E_{m+2}+\cdots$ is contained in the interval $0<x<1/(2m+1)!$, so that

$$(3) \qquad \left| I(E_{m+1} + E_{m+2} + \cdots) \right| < m(E_{m+1} + E_{m+2} + \cdots) < \frac{1}{(2m+1)!}.$$

It follows from (2) and (3) that

$$(2m)!\left| C_{(2m)!} \right| > \frac{2m}{\pi} - \frac{1}{2m+1},$$

which proves that the left side of the last inequality approaches ∞ as $m \to \infty$.

Therefore, a negative answer is indicated for the proposed question.

Also solved by S. P. Lloyd, Karl Zeller, and the proposer.

Gaussian Divisibility

4750 [1957, 509]. *Proposed by Leonard Carlitz, Duke University*

Find the denominator of

$$\binom{i}{m} = \frac{i(i-1)\cdots(i-m+1)}{m!}$$

where $i=\sqrt{-1}$ and the fraction is reduced to lowest terms in the field $R(i)$.

Solution by the proposer. Put $m!=2^ePQ$, where the primes dividing P are of the form $4k+1$, the primes dividing Q of the form $4k-1$, and

$$e=\left[\frac{m}{2}\right]+\left[\frac{m}{4}\right]+\cdots.$$

We shall show that the denominator of $\binom{i}{m}$ is

$$(1+i)^{2e-[m/2]}Q. \tag{1}$$

1. Let q be a rational prime $\equiv 3 \pmod 4$, so that q remains a prime in the Gaussian field $R(i)$ and the product $i(i-1)\cdots(i-m+1)$ is not divisible by q. Consequently the denominator of $\binom{i}{m}$ contains Q.

2. We have $2=-i(1+i)^2=i(1-i)^2$. A number $i+r$, where r is a rational integer, is divisible by $1+i$ if and only if $N(i+r)=1+r^2$, the norm of $i+r$, is even; but for r odd, $1+r^2\equiv 2 \pmod 4$, so that $i+r$ is not divisible by $(1+i)^2$. Thus $i(i-1)\cdots(i-m+1)$ is divisible by exactly $(1+i)^{[m/2]}$ and therefore the denominator of $\binom{i}{m}$ is divisible by $(1+i)^{2e-[m/2]}$ in accordance with (1).

3. It remains to show that

$$i(i-1)\cdots(i-m+1)\equiv 0 \pmod P. \tag{2}$$

Let p denote a rational prime $\equiv 1 \pmod 4$. Then $p=\pi\pi'$, where π, π' are distinct primes of $R(i)$. Choose a so that $a^2+1\equiv 0 \pmod p$; then $\pi \mid (i-a)$; we may suppose $1<a<p$. We recall that the numbers $0, 1, \cdots, p^n-1$ constitute a complete residue system (mod π^n) for every $n\geqq 1$. Thus in the sequence $i, i-1, \cdots, i-m+1$ there are $[m/p]$ multiples of π, $[m/p^2]$ multiples of π^2, and so on. Consequently $i(i-1)\cdots(i-m+1)$ is divisible by p^f, where

$$f=\left[\frac{m}{p}\right]+\left[\frac{m}{p^2}\right]+\cdots.$$

Similarly, starting with $\pi' \mid i-p+a$, we infer divisibility by π'^f. This completes the proof of (2).

Geometric Means

4752 [1957, 510]. *Proposed by M. S. Klamkin, AVCO Research and Development, Lawrence, Mass.*

Determine a set of n distinct, nonzero terms such that their geometric mean is the geometric mean of their arithmetic and harmonic means.

Solution by Emil Grosswald, University of Pennsylvania. Let S_j be the jth fundamental symmetric function of the n terms $a_1, \cdots, a_n$. Then their harmonic, geometric and arithmetic means are

$$H=nS_n/S_{n-1}, \qquad G=S_n^{1/n}, \qquad A=S_1/n,$$

respectively, and the condition $G^2=HA$ of the problem becomes

(1) $$S_{n-1} = S_1 S_n^{1-2/n},$$

It is therefore sufficient to take as $a_1, \cdots, a_n$ the roots of

(2) $$x^n - S_1 x^{n-1} + \cdots + (-1)^{n-1} S_{n-1} x + (-1)^n S_n = 0,$$

with arbitrary $S_2, S_3, \cdots, S_{n-2}$ and any S_1, S_{n-1}, S_n which satisfy (1). In order to have all terms different from zero it is sufficient to take $S_n \neq 0$; and $a_i \neq a_j$ for $i \neq j$ is assured if the coefficients of (2) are so chosen that its discriminant does not vanish.

Also solved by A. P. Boblétt, V. E. Hoggatt, W. S. Lawton, J. B. Muskat, Marlow Sholander, Michael Skalsky, and the proposer.

Editorial Note. Several explicit sets were proposed. The simplest are: (i) n successive terms of any geometric progression, (ii) $[n/2]$ distinct pairs of reciprocals, with the addition of the element 1 in case n is odd.

Integers of Special Form

4753 [1957, 596]. *Proposed by J. K. Senior, University of Chicago*

Are there any integers larger than 31 which can be represented in more than one way by the form $(a^m-1)/(a-1)$, a and m integers and $m>2$?

Comment by A. Makowski, Warsaw, Poland. We have

$$8191 = \frac{2^{13}-1}{2-1} = \frac{90^3-1}{90-1}.$$

This example was proposed by R. Goormaghtigh (*L'Intermédiaire des Math.*, 1917, pp. 88, 1530) who stated that for $A<1{,}000{,}000$ the only solutions are 31 and 8191. A reference is given in Dickson, *History of the Theory of Numbers*, vol. 2, p. 703. See also *Mathesis*, 1957, p. 327.

Also solved by W. H. Benson and M. S. Klamkin.

RECENT PUBLICATIONS

EDITED BY RICHARD V. ANDREE, University of Oklahoma

All books for review should be sent directly to R. V. Andree, Department of Mathematics, University of Oklahoma, Norman, Oklahoma, and not to any of the other editors or officers of the Association.

Mathematics and Plausible Reasoning. By G. Pólya. Princeton Univ. Press, 1954. Vol. I, $5.50. Vol. II, $4.50. Both $9.00.

Vol. I. *Induction and Analogy in Mathematics.* xvi+280 pp.

The author has gathered an interesting list of problems which substantiate his thesis that mathematics requires imaginative guessing as well as deductive

argument. "You have to guess a mathematical theorem before you can prove it; you have to guess the idea of the proof before you carry through the details." His examples are chosen chiefly from arithmetic, algebra, and elementary geometry, but there are some sections which use the notions of differential and integral calculus.

Particularly interesting are the problems in maxima and minima which are solved without calculus. The author resists the temptation to give rules of thumb for handling various types of problems. Accordingly, the experienced teacher may be impatient with the author's leisurely style. Yet it is precisely because blind alleys have been deleted from published mathematics that makes it seem unmotivated to the student reader.

Vol. II. *Patterns of Plausible Inference.* x+190 pp.

Apparently the second volume is intended as a guide to any student who wishes to learn how to use induction and analogy in reasoning. This reviewer confesses disappointment in the result. The author belabors the point that creative reasoning is not always deductive. He spells out the precise pattern of guessing which can lead to the solution of a variety of additional problems. The moral seems to be "Go thou and do likewise," but, as hindsight is often clearer than foresight, it is doubtful whether creativity in mathematics would be encouraged by following this (or any other) set of rules.

The treatment of probability is superficial. Thus a linguistic example treats all letters as equally probable, and ignores borrowing. No attempt is made to use information theory to measure credibility, and the author supposes that no quantitative theory of plausible reasoning may be possible. The analogy with many-valued logics is also ignored. In conclusion the author says the exposition is "principally for students, but perhaps also useful to some teachers," but not to one who tries to make students understand things he does not understand himself.

ARTHUR BERNHART
University of Oklahoma

Elementary Differential Equations. By William Ted Martin and Eric Reissner. Addison-Wesley, Reading, 1956. xii+260 pp. $5.50.

Before evaluating this text, let us set up standards with which it may be compared. A textbook for a first course in differential equations which is intended primarily for students of science and engineering might be selected on the basis of the following requirements.

I. *Matters of principle:*

1. The text should give a treatment of practical value.
2. It ought to be mathematically sound.
3. It has to be understandable for a student who has just squeezed through calculus.

The present, and some other recent books, come as close to passing this first test as one can reasonably expect. Hence let us formulate a second set of requirements, and really look at what's in the book.

II. *Contents.* The text should contain:

1. A chapter on the setting up of differential equations.
2. No chapter on first-order equations of special types for which there happen to be age-old tricks leading to solutions in closed form.
3. Several chapters scattered throughout the book which consist of the following commercial, printed in bold-faced capitals across two adjoining pages:

   "LEARN A LITTLE GERMAN AND LOOK FOR THE SOLUTION OF ANY GIVEN DIFFERENTIAL EQUATION IN THE SOURCE BOOK BY

   E. KAMKE

   DIFFERENTIALGLEICHUNGEN. LÖSUNGSMETHODEN UND LÖSUNGEN. I GEWÖHNLICHE DIFFERENTIALGLEICHUNGEN. LEIPZIG 1944."

4. Several chapters on approximate solution of the equation $y'=f(x, y)$. The treatment should include graphical methods (isoclines), the methods of power series, successive approximation and difference equations, and an existence and uniqueness theorem. At this stage exact solutions may be given for the types "variables separable" and "linear."
5. A chapter on linear equations of second and higher order with constant coefficients.
6. A chapter on second-order linear equations with simple analytic coefficients, treating series solutions and the behavior of solutions near (regular) singular points, including infinity.
7. A chapter dealing with linear systems. It is desirable to introduce the student to vector-matrix notation here. This chapter is the proper place for the introduction of "variation of parameters."
8. No chapter on applications where the main difficulties are in the applications.
9. No last-chapter attempt to treat the whole theory of partial differential equations, including Fourier series and other orthogonal expansions.

The present book scores much higher on test II than many other recent texts. It gives an outstanding treatment of power series methods, both for first-order nonlinear and for higher-order linear equations. Nevertheless there are bad letdowns when it comes to points 3 and 9 above. The authors meet points 2 and 7 only part way.

There are some finer points on which the reviewer likes to comment.

The authors define a solution as a relation $g(x, y)=0$. The reviewer feels that this is too much of a concession to the special case of an exact equation of the first order. He prefers to use the word function and to tie a solution to an interval.

The reviewer likes to solve the equation $y'=f(x)/g(y)$ by writing $g(y)y'=f(x)$ rather than $g(y)dy=f(x)dx$. A similar remark applies to the exact equation: avoidance of differentials tends to improve the understanding.

The power series method is used to investigate the behavior of solutions of linear equations near a finite singularity, but nothing is done about behavior near infinity.

In dealing with linear differential equations with constant coefficients the authors first give the method of guessing exponential solutions. They follow it by the Laplace transform method which reduces the differential equation to an algebraic equation. They do not mention the operator method of D's which is more elegant than guessing and has the advantage over the Laplace transform method of being independent of special assumptions about the growth of right hand member and solutions.

J. KOREVAAR
The University of Wisconsin

Quality Control and Applied Statistics Abstracts (Including *Operations Research*). Editors: Robert S. Titchen, Arnold J. Rosenthal, Bruce Bollerman, Frank Nistico. Interscience, New York. One volume of approximately 1000 pages in 12 issues annually, starting June 1956. $60.00 per year.

Articles considered by the editors to contain new information of lasting interest to the technical worker in Quality Control, Applied Statistics or Operations Research are abstracted each month from approximately 400 domestic and foreign journals. Each abstract shows fully the important contributions of the article abstracted including such items as formulas, illustrations, tables and references. The abstracts are published in loose-leaf form on strong $6\frac{1}{2}\times 9\frac{1}{2}$ inch sheets of white paper. For filing convenience each abstract is given a code numeral indicating the type of treatment and a letter indicating the field of application. (Under type of treatment the main divisions are Statistical Process Control, Sampling Principles and Plans, Management of Quality Control, Mathematical Statistics and Probability Theory, Experimentation and Correlation, Managerial Applications, Measurement and Control Instrumentation.) Issue 1 of vol. 1 lists the journals reviewed and the classification of abstracts.

Issues 1 and 2 of vol. 1 and issue 2 of vol. 2 were found to contain xvi+80 pp. including 37 abstracts from 28 journals, 86 pp. including 43 abstracts from 30 journals and 96 pp. including 39 abstracts from 21 journals, respectively.

This is an essential journal for all workers in the fields that it surveys.

JOHN C. BRIXEY
University of Oklahoma

Grundprobleme der Mathematischen Theorie Elektromagnetischer Schwingungen. By Claus Müller. Springer, Berlin, 1957. ix+344 pp. DM 49.60.

Experience indicates that the classical electromagnetic radiation field is essentially uniquely determined by the nature and distribution of the matter

present. This poses a formidable mathematical problem: given the mathematical equations governing the electromagnetic field, to specify adequate mathematical boundary conditions that will correspond to reasonable physical situations, and to show that under these various conditions the corresponding solutions exist and are unique.

In the admirable book under review, Professor Müller, who has made important contributions towards the solution of this problem, shows how it is solved for the case of electromagnetic oscillations when the boundary conditions do not involve the time; and in so doing he presents this aspect of the mathematical theory of electromagnetic oscillations in a state of completion and elegance comparable to that of potential theory. He assumes, of course, that the reader is mathematically sophisticated; for instance, in his preparatory chapter "The Fundamental Concepts of Vector Analysis" he plunges immediately into the problem of redefining differential vector operators as limits of multiple integrals so that they will be applicable to nondifferentiable operands.

After some preliminary orientation, Professor Müller treats the classical case of scalar oscillations in detail, presenting it as a complete entity while using it as an occasion to develop ideas needed for the more difficult vector case. The full presentation of the solution of the latter case for both homogeneous and nonhomogeneous media occupies the major portion of the book and is the outstanding achievement that makes the book unique.

Professor Müller is a gifted expositor. His book is notable not only for its mathematical importance and its high standard of rigor but also for its exceptional clarity.

BANESH HOFFMANN
Queens College

An Introduction to Genetic Statistics. By Oscar Kempthorne. Wiley, New York, 1957. xvii+545 pp. $12.75.

This is a large and exceptionally complete volume, presenting in detail the various statistical tools which the research geneticist needs in his work. Practical genetic examples are provided under each topic.

The mathematics involved in the presentation do not go beyond algebra and the differentiation of quadratic forms, yet the topics considered are abstruse enough that a good fundamental understanding of statistical methods and a thorough working knowledge of genetics are both essential prerequisites for the use of the book. The opening chapter is a highly condensed presentation of elementary probability. Then comes a chapter devoted to an equally strong distillation of the principles of genetics.

Individual topics follow, each treated in highly concentrated form. They include among others selection, inbreeding, tests of genetic hypotheses, estimation of genetic parameters, planning of genetic experiments, use of matrices, analysis of variance, multiple regression, path coefficients, correlations and equilibria. Each topic is discussed in relation to various genetic complications

and chromosomal aberrations.

The author has included not only expositions of the works of others, but also considerable original work on some of the more difficult phases of genetics. As concentrated as the presentation is, it covers more than 500 pages. The research worker in genetics will find the book indispensable for the statistical treatment of very specific problems, even the most recondite.

LAURENCE H. SNYDER
The University of Oklahoma

Report on A Survey of Training and Research in Applied Mathematics. By F. J. Weyl, Investigator. Monograph No. 1, Society for Industrial and Applied Mathematics, Philadelphia, 1956. $2.00.

The objectives of this Survey were:

(a) to determine the nature and extent of research in applied mathematics currently being carried forward in universities, government agencies, and industrial establishments;
(b) to estimate the nature and extent of the unrealized potentialities of mathematics in applied contexts;
(c) to suggest the kind and character of further research in this field which is needed in the national interest;
(d) to consider the kind of training in applied mathematics which is now available in the United States; and
(e) to relate this information to the need for trained mathematicians in future years.

These objectives were achieved through (i) an investigating committee, (ii) a questionnaire, (iii) conferences. Reports of the conferences (iii) were published by the National Science Foundation and also in this MONTHLY, vol. 61, No. 7, Part II, 1954.

The report under review, which deals with (i) and (ii) above, is in two parts: I, Summary and Recommendations; and II, Review of Current Developments. In I, among other things, it is recommended that mathematicians in industry and government be invited to give courses in universities, and that university faculty members who are able to do so should give courses in applied mathematics. Part II discusses the evolution of the "Scientific Substance," and gives a review of applied mathematics activities.

C. O. OAKLEY
Haverford College

Introductory College Mathematics. By Robert W. Wagner. McGraw-Hill, New York, 1957. 430 pp. $5.50.

There is nothing very unusual about the table of contents. We find Numbers, Equations and Inequalities, four chapters on elementary functions, one each on

Trigonometry, Conics, and Calculus (of polynomial functions), and finally some material on Interest, Annuities, Statistics, and Probability.

The text is long-winded, sometimes "folksy," and often merely absurd. We may classify some of the absurdities as (a) pointless, (b) confusing, (c) misleading, and (d) false.

(a) Pointless. Addition and multiplication of "counting numbers" are interpreted as taking unions and cartesian products of sets; the laws of arithmetic are "proved" from this interpretation. The student is then expected to apply this new insight to problems like (p. 12): "A line segment is 7 feet long. Given that there are 12 in. in a foot, how many inches long is the segment? Use the set of inches to justify your answer."

(b) Confusing. On page 13 we read: "Theorem. The number of primes is not a counting number." The first use of "number" here is not covered in the author's Chapter 1 catalogue: Counting Numbers, Negative Numbers, Rational Numbers, Real Numbers, Complex Numbers, and Approximate Numbers.

(c) Misleading. On page 45, "relation" means "equation or inequality." On page 81, a relation is a set-valued function; this definition is fortunately not pushed very far, and is violated on page 83, where the graph of an inequality is implied to be a union of lines rather than a collection of them.

(d) False. On page 5, "most practical sets can be put into one-to-one correspondence with a subset of the counting numbers," On page 140, concerning the functions $f(x)=kx^m$, "Since the uses of these functions with negative values of k are very uncommon. . . . "

As for general absurdity, a good example, again concerning $f(x)=kx^m$, is "A second property of these functions and the one which accounts for their wide use is that if the value of x is doubled or tripled the value of the image is multiplied by 2^m or 3^m." Another example is the naming of twelve, rather than the usual six, inverse trigonometric functions (p. 222).

The reviewer has the impression that the author is trapped by his desire to write an introduction to mathematical thinking rather than to mathematical methods. He does neither. Even the problems illustrate only the most routine calculations. Nonroutine problems are advertised by an asterisk (see Preface); there are precisely two such problems (pp. 215 and 364).

WALTER RUDIN
University of Rochester

Calculus. By Jack R. Britton. Rinehart, New York, 1957. xiv+584 pp. $6.50.

This is one of many calculus texts, and one of the better. The author's stated purpose—"to present an introductory course in the calculus as simply as possible but with due regard for the modern requirements of rigor"—has been attained at a level of exposition suitable for both college students and advanced-standing high-school students.

The use of tables of numerical values for velocity and tangent problems in the first chapter provides an excellent basis for the later development of limits and derivatives. The scope of the book is traditional, with derivatives introduced on page 22, differentials on page 93, integrals on page 108, infinite series on page 316, approximate integrals on page 388, partial differentiation on page 396, multiple integrals on page 434, and differential equations on page 467. There is a table of 105 integrals. The appendix includes 22 infinite series for reference and about 70 curves with their equations. The book is self-contained with respect to tables. There are ample numerical exercises and problems; answers for most odd-numbered problems are included at the back of the book.

Some instructors will want to emphasize the concept of "range" when variables are first introduced, to consider some of the applications of the integral earlier, and to consider approximate integration earlier. Such modifications should not be difficult. Even if considered necessary, their inconvenience would be more than compensated for by the "Review and Discussion Questions" at the end of most chapters and the excellent exposition throughout the book.

BRUCE E. MESERVE
State Teachers College
Montclair, New Jersey

Mathematical Analysis. By Tom M. Apostol. Addison-Wesley, Reading, 1957. xii+553 pp. $9.00.

This book treats the topics which usually fall under the heading of "Advanced Calculus." It includes rigorous proofs of many of the theorems which are usually considered too difficult for an advanced calculus text, but too elementary for a course in function theory. In the author's words, "the book helps to fill the gap between elementary calculus and advanced courses in analysis. More important than this, it introduces the reader to some of the abstract thinking that pervades modern mathematics."

The first chapter, "The Real and Complex Number System," forms an excellent introduction. Elements of set theory are presented in the next two chapters. The remaining chapters include a discussion of limits and continuity, differentiation, applications of partial differentiation, functions of bounded variation, connectedness, the Riemann-Stieltjes integral, multiple and line integrals, vector analysis, infinite series and products, Fourier series and integrals, and an introduction to the theory of functions of a complex variable.

The appearance of the book is pleasing. Theorems and definitions are stated in italics. In addition to the usual index, the book contains an index of special symbols.

The exercises which appear at the end of each chapter are unusually well chosen; many of them will present a real challenge to the better students. Lists of references for further study conclude many of the chapters.

Because of the difficulties involved when many omissions are made from a

text, this book would seem most suitable for students who have had some introductory work in advanced calculus. For such classes it should prove to be excellent both as a text and as a reference book.

F. M. MEARS
The George Washington University

Theorie der Beugung Elektromagnetischer Wellen. By W. Franz. Springer, Berlin, 1957. iv+123 pp. DM 21.60.

If the title of this book led one to seek in it a comprehensive exposition of the theory of the diffraction of electromagnetic waves, one would be disappointed by its lack of completeness. But the book could hardly aim at completeness in view of its mere hundred-odd pages, and is more aptly described as a monograph than as a treatise. It takes for granted existence and uniqueness theorems, and is principally, though by no means wholly, concerned with presenting a detailed, self-contained account of the method of solution by means of the Watson transformation, especially as applied to the cylindrical case for which Professor Franz himself first gave the solution. The Watson transformation is a powerful device whereby certain series can be transformed into other series that converge far more rapidly. The author treats the subject with authority and insight, and in ample detail, giving careful discussions, for example, of the paths of integration of the complex integrals that yield the Watson transformations for the cylindrical and spherical cases, and explaining clearly the significance of the residue waves as the so-called creeping waves.

These items are contained in the central section of the book. They are preceded by a preliminary section dealing with such necessary matters as Green's functions and dyadics, and boundary conditions; and they are followed by a section dealing with diffraction by objects having edges. This final section treats an assortment of special topics, from the Sommerfeld wedge theory to the recent Braunbek method, and covers, with a few notable exceptions, the principal problems whose solutions are known.

The book is up-to-date both in method and outlook. Despite an evident pressure of space, Professor Franz has illuminated it with frequent interpretative insights. It will be warmly welcomed by people working in the field, and should be of more than passing interest to other mathematicians.

BANESH HOFFMANN
Queens College

Numerical Analysis. By Kaiser S. Kunz. McGraw-Hill, New York, 1957. xv+381 pp. $8.00.

This book, based on graduate lecture courses at the Harvard University Computation Laboratory, is designed to "acquaint the student with the best procedures available for obtaining numerical solutions to problems arising in applied mathematics," with special attention to differential and integral equa-

tions. Eleven chapters discuss interpolation, in one and two variables, numerical differentiation and integration, the start and continuation of step-by-step methods for ordinary differential equations, and some methods for solving elliptic, parabolic and hyperbolic partial differential equations and Volterra and Fredholm integral equations. Four other chapters deal with the roots of equations and particularly polynomials, the summation of series, and the solution of linear algebraic equations and the inversion of matrices. A short appendix discusses simple error analysis.

There are also some surprising omissions: for example, the "cross-mean" iterative interpolation method of Aitken, the work of Comrie and others on subtabulation and the use of modified differences, the valuable implicit method for parabolic equations, for hyperbolic equations the method of integrating along characteristics, and for algebraic equations the concept of ill-conditioned matrices. Most important, there is no discussion of the eigenvalue problem, for either matrices or differential operators. This is surely an important problem in applied mathematics, not to be dismissed as a peripheral subject along with "smoothing, least-square approximation and harmonic analysis."

The material, however, is well presented, has little "advanced" mathematics, and is interesting and instructive to both student and expert. The chapter on integral equations is particularly good, that on bivariate interpolation more readable than most such expositions, and there is an interesting extension of the lozenge diagram for differentiation and integration. Remainder terms are derived for every formula, there is a useful discussion of error accumulation in step-by-step integration, and both worked examples and exercises for students are included in every chapter. The student, while undoubtedly benefiting from a knowledge of this book, will also have many questions to put to his supervisor.

L. Fox
Oxford

An Introduction to Diophantine Approximation. By J. W. S. Cassels. Cambridge Tracts in Mathematics and Mathematical Physics No. 45, Cambridge University Press, New York, 1957. x+166 pp. $4.00.

This tract presents basic techniques and outstanding results of Diophantine approximation in a way such that an undergraduate senior or first-year graduate student who has a knowledge of the elements of number theory should be able to read it. However, one chapter requires the elements of Lebesgue theory and another chapter requires the elements of algebraic number theory. Material needed from the geometry of numbers appears in the appendixes.

The first chapter on homogeneous approximation introduces the continued fraction process, equivalence, application to approximations and simultaneous approximation. This is followed by a study of indefinite binary quadratic forms, Markoff forms, the Markoff chain of forms and the Markoff chain for approximations. Chapter III introduces inhomogeneous approximation with a study of

the 1-dimensional case (two Minkowski theorems) and simultaneous approximation (Kronecker's theorem). The chapter on uniform distribution considers uniform distribution of linear forms and Weyl's criteria and consequences. The chapter on transference theorems studies transference between two homogeneous problems and application to simultaneous approximations, transference between homogeneous and inhomogeneous problems with application to inhomogeneous approximation, a quantitative Kronecker's theorem and successive minima. The sixth chapter is concerned with Roth's theorem and rational approximation to algebraic numbers. Chapter VII on metrical theory assumes a knowledge of Lebesgue theory. The concluding chapter studies theorems of Pisot, Vijayaraghavan and Salem on Pisot-Vijayaraghavan numbers. Each chapter has a careful introduction and is concluded with an illuminating section of notes. The three appendixes discuss bases in certain modules, tools from the geometry of numbers and Gauss's lemma. An excellent five-page bibliography is included.

JOHN C. BRIXEY
University of Oklahoma

International Series of Monographs in Pure and Applied Mathematics. Vol. 1. *An Introduction to Algebraic Topology.* By A. H. Wallace. Pergamon, New York, London, Paris, 1957. 198 pp. $6.50.

Homology theory has been given a beautiful axiomatization by Eilenberg and Steenrod in their book "Foundations of Algebraic Topology." In the preface to this volume the authors state "Homology theory is a transition (or function) from topology to algebra. It is this transition which is axiomatized."

The present volume serves as a motivation for this transition. The first third of the book gives the basic topology needed before the appearance of any algebra. A topological space is defined by means of the collection of all neighborhoods of each of its points, and these neighborhoods need not be open sets. It turns out that a neighborhood of a point p is any set containing any open set which contains p. The reviewer feels that "vicinity" is a better choice of name for such a set than "neighborhood." The other novelty about this topological discussion is the successful definition of arcwise connected spaces without defining arcs.

Chapters IV to IX, the remainder of the text, discuss the fundamental group and develop the theory of homology sequences. Chapters VI, VII, VIII prove the Homotopy, Excision, and Exactness Axioms of Eilenberg and Steenrod for the spaces discussed by the author. It is unfortunate that the Exactness Axiom comes so late in the book (p. 153) that no effective use is made of the powerful tool of commutativity diagrams.

It is a disappointment in an introductory text such as this that the Jordan Curve theorem, while assumed, is never explicitly mentioned. Practically all of the text consists of building up machinery which can be used in later courses, *e.g.* in a course based on Eilenberg and Steenrod. Important application will be

discovered by the student only if he digs into the numerous problems. One such application (Problem 2, p. 167) is the fixed-point property for solid r-dimensional Euclidean spheres. Problem 7, page 54, and Problem 1, page 61, contain errors.

DICK WICK HALL
Harpur College

The Teaching of Mathematics. By the Incorporated Association of Assistant Masters in Secondary Schools, Cambridge University Press, New York, 1957. ix+231 pp. $3.00.

This volume gives a complete and clear picture of the mathematics program in the grammar schools of England. The subject matter is outlined in detail, and the spirit and philosophy of instruction emerge in every chapter. The program reveals a true English conservatism that doggedly plods ahead to necessary reform. There is a great change from nineteenth-century sequential Euclidean geometry. The algebra program remains typically that of the English schools. Perhaps unique in this book is a summary of expectations from pre-secondary school study of mathematics and the large segment of space devoted to pedagogical matter. Among the latter are procedures for inculcating a love for mathematics, the organization of the class lesson, the role of questioning, tests and examinations; the use of audio, visual and other aids; the role of a mathematics homeroom or laboratory, a mathematics library and its use and the role of history in mathematics.

The teaching of statistics as a new subject (since 1948) reflects an influence of Yule and Kendall in determining the topics taught. Perhaps one novel characteristic is that of a sixth form which can occupy one, two or three years of study before the student continues to the university. The primary change, in point of view of organization, is the breakdown of barriers between arithmetic, algebra, geometry and analysis in order to achieve unity. It is doubtful that the proposed syllabus will achieve this unity. To a reader who is concerned with the teaching of mathematics in the U. S., there cannot help but come the feeling that the problems posed in this book are those with which we have been struggling in our country during the last 20 years and from which we have now moved forward.

HOWARD F. FEHR
Columbia University

BRIEF MENTION

Offerings and Enrollments in Science and Mathematics in Public High Schools, 1956. By Kenneth E. Brown and Ellsworth S. Obourn. U. S. Department of Health, Pamphlet No. 120, 1957. 44 pp. $0.25.

It is encouraging to note that the percentages of students taking science and mathematics courses have increased between 1954 and 1956, as a comparison of the study based on the 1954 data shows (see review this MONTHLY,

vol. 64, 1957, p. 130). The data are well presented. Not only national averages, which are, after all, somewhat meaningless, but also regional data are given, with the United States broken into nine regions. The national average shows that only eight per cent of the pupils in the twelfth grade are now in high schools which do not offer trigonometry, intermediate algebra, solid geometry or other comparable mathematics. However, the regional percentages range from 0.4 per cent to 19.1 per cent. This inexpensive little booklet should prove helpful to colleges and universities planning to revise their mathematics requirements in the realm of remedial and freshman mathematics.

Puzzle Math. By George Gamow and Marvin Stern. Viking, New York, 1958. 119 pp. $2.50.

A collection of 32 puzzles or brain twisters, some old, some new, but all charmingly presented in the authors' inimitable style. A pedant might quibble over the phrase "three times smaller" (p. 61), but this is picayune.

Mathematics Charts from the Dawn of History to 1950 A.D. By L. E. Christman and L. S. Overeem. Christman, Yorkville, Illinois. 13 pp. $1.50.

A set of charts which could serve as a framework for a talk on the history of mathematics. One may be surprised to see an ordinary desk calculator under the heading, "High Speed Computing Machines."

An Introduction to Scale Coordinate Physics. By William Bender. Burgess, Minneapolis, 1958. ix+340 pp. $7.50.

NEWS AND NOTICES

EDITED BY LLOYD J. MONTZINGO, JR., University of Buffalo

Readers are invited to contribute to the general interest of this department by sending news items to L. J. Montzingo, Jr., Mathematical Association of America, University of Buffalo, Buffalo 14, New York. Items must be submitted at least two months before publication can take place.

NATIONAL SCIENCE FOUNDATION SPONSORS CONFERENCE ON NATIONAL PROBLEMS IN MATHEMATICS

A special conference to consider national problems of research and training in mathematics was held at the University of Chicago, February 20 and 21, under the auspices of the National Science Foundation. Mathematicians representing all sections of the country held sessions on the following topics: (1) the need for centers of advanced study, (2) federal support for mathematical research in the universities, (3) texts for high school mathematics.

NOTES ON ANNUAL MEETING OF THE ASSOCIATION

Notes on the three program sessions of the forty-first annual meeting of the Mathematical Association of America have been prepared by Mr. Frank Kocher. Copies of these notes are available to members of the Association on request to Mr. Kocher at the address: Department of Mathematics, Pennsylvania State University, University Park, Pennsylvania.

INTENSIVE COURSE IN OPERATIONS RESEARCH

An intensive course in Operations Research and Systems Engineering for business, industrial and government personnel was offered by the School of Engineering of the Johns Hopkins University from June 9 through June 20, 1958, at the Homewood Campus of the University. Introductory lectures were given by the Dean of the School of Engineering and the Directors of the University's Applied Physics Laboratory and Operations Research Office. These were followed by numerous case studies and expository lectures on Operations Research, Systems Engineering, Cost Data, Models, Human Engineering, Data Processing, Simulation, Information Theory, Quality Control, Design of Experiments, Game Theory, Flow Graphs, System Dynamics, Inventory Systems, Waiting Lines, Symbolic Logic, Stability and Linear Programming.

PRINCETON PROGRAM TO MODERNIZE COLLEGE PREPARATORY MATHEMATICS

A long-range approach to the objective of evolving a truly modern curriculum in college preparatory mathematics is being undertaken in Princeton, New Jersey, by the school boards of five different municipalities working in close cooperation with mathematicians associated with Princeton University and the Princeton-headquartered Educational Testing Service. Based upon nearly a year of discussions on the part of a Curriculum Revision Committee, brought into being by the Princeton Borough and Township Schools and more recently expanded to include three bordering municipalities, the newly-established program calls for a long, hard look at mathematics—from beginners' arithmetic to college calculus—as an integral part of the education of any future citizen.

The first step consists of a fifteen-week series of one-hour classes which are designed to give mathematics teachers in public elementary and secondary schools "added insight into the nature of mathematics as a creative endeavour in twentieth-century civilization." This orientation program, open on a voluntary basis to all mathematics teachers in the five cooperating school systems, has been divided into two sections, "advanced," meaning "algebra and beyond," and "elementary" for teachers in Grades IV through VII.

The orientation classes, continuing through June, will be supplemented next fall by a year-long "curriculum for teachers," which will be largely concerned with course-material in special areas, such as algebra, plane and solid geometry and trigonometry, and the relationships between the teachers' individual courses and new conceptions in mathematics.

In the near future, beyond the 1958–59 plan of study for teachers, who will also be urged to participate in summer institutes in mathematics, lies a revamped curriculum that will give students a view of contemporary mathematics as a whole and will enable qualified students to take the equivalent of an added year of advanced mathematics at the high-school level.

The present organization of the "Princeton Mathematics Program" has been carried forward by two expert parent-advisers to the schools' Curriculum Revision Committee, Dr. Marion G. Epstein, a member of the Mathematics Section in the Educational Testing Service's Test Development Division, and Professor Albert W. Tucker, chairman of the Princeton University's Department of Mathematics.

PERSONAL ITEMS

Professor G. B. Price, of the University of Kansas and Professor B. W. Jones of the University of Colorado have been appointed official delegates of the Association to the International Congress of Mathematicians, to be held in Edinburgh in August, 1958.

Professor G. B. Price represented the Association at the inauguration of Richard Aubrey McLemore as President of Mississippi College on March 19, 1958.

Professor M. F. Rosskopf of Teachers College, Columbia University, has received a Fulbright Award to lecture at the Pedagogisk Seminar, University of Oslo, for the year 1958–59.

Mr. M. T. Austin, Rutgers University, has accepted a position as mathematics analyst for the Chrysler Missile Division, Detroit, Michigan.

Mr. W. E. Beeman, North Texas State College, has been appointed Associate Professor at Arlington State College.

Assistant Professor W. S. Bishop, Howard College, has been promoted to Associate Professor.

Mr. C. K. Bradshaw, University of Nevada, is now a graduate assistant at Stanford University.

Dr. Louis DeBranges, Cornell University, has been appointed Assistant Professor at Lafayette College.

Mr. R. S. Dick, Columbia University, has been appointed Instructor at Queens College of the City of New York.

Mr. G. E. Duncan, Georgia Institute of Technology, has accepted a position as associate mathematical engineer for Lockheed Aircraft Corporation, Marietta, Georgia.

Miss Angeline W. Evans, Agnes Scott College, has accepted a position with the First National Bank of Atlanta, Georgia.

Pfc. P. J. Finn, St. John's College, Brooklyn, is now serving at the Combat Development Experimentation Center, Fort Ord, California.

Professor Michael Golomb, Purdue University, is on leave during 1957–58 and is a member of the Mathematics Research Center, University of Wisconsin.

Mr. R. G. Green, CONVAIR, San Diego, California, has accepted a position as dynamics engineer.

Dr. B. R. Levy, Institute of Mathematical Sciences of New York University, has been appointed Research Associate.

Mr. W. J. Lyche, North American Aviation, Los Angeles, California, has been appointed Assistant Professor at Long Beach State College.

Mr. W. J. Mays, Imperial Life Insurance Company, Asheville, North Carolina, has accepted a position as actuary with Western & Southern Life Insurance Company, Asheville.

Assistant Professor Ralph Playfoot, Lafayette College, has been promoted to Associate Professor.

Associate Professor T. E. Rine, Illinois State Normal University, has been promoted to Professor.

Mr. R. E. Schlea, Ohio Northern University, has been appointed Assistant Professor at Baldwin-Wallace College.

Mr. W. L. Shepherd, Texas Western College, has been appointed Assistant Professor.

Assistant Professor Peter Terwey, Jr., Davidson College, has been appointed Associate Professor at Lamar State College of Technology.

Professor L. F. Tolle, St. John's University, New York, has been appointed Chairman, Department of Mathematics.

Mr. V. D. Turner, Mankato State Teachers College, has been appointed Assistant Professor.

Dr. Jack Warga, Electro-Data Corporation, Pasadena, California, has accepted a

position as senior staff scientist with AVCO Manufacturing Corporation, Lawrence, Mass.

Associate Professor L. A. Warwick, Wesleyan College, has been appointed Head, Department of Mathematics at William Jennings Bryan University.

Mr. A. M. White, University of Santa Clara, has been appointed Assistant Professor.

Mr. R. S. Wolfe, Northwestern University, has been appointed Assistant Professor at Rollins College.

Miss Elaine G. Yodice, University of Wisconsin, has been appointed Instructor at the University of Massachusetts.

Mrs. Grace A. deForest died on April 5, 1958 after a brief illness. For the past nine years she has been in charge of the Buffalo office of the Association. She has served the Association well and faithfully and will be greatly missed.

Professor Emeritus Floyd Field, Georgia Institute of Technology, died February 14, 1958. He was a charter member of the Association.

Assistant Professor A. S. Hendler, Rensselaer Polytechnic Institute, died June 11, 1957.

Mr. E. H. Koch, Jr., retired, died January 26, 1958. He was a charter member of the Association.

Sister Ann Elizabeth, Professor and Registrar, St. Mary College, Kansas, died May 14, 1957. She had been a member of the Association for 23 years.

THE MATHEMATICAL ASSOCIATION OF AMERICA

Official Reports and Communications

NEW MEMBERS

Professor H. M. Gehman, Secretary-Treasurer, announces that the following 96 persons have been elected to membership by the Board of Governors on applications duly certified.

HERBERT L. ARNOLD, Freezer-attendant, Campbell Soup Co., Chicago, Illinois.

DENVER BAILEY, M.S. (Kentucky) Teacher, St. Petersburg Junior College.

NICHOLAS S. BARKAS, M.A. (Boston) Professor, Varuakion Graduate School, Athens, Greece.

LOUIS J. BARRON, M.A. (Michigan) Instructor, St. Petersburg Junior College.

WILLARD E. BAXTER, Ph.D. (Pennsylvania) Asst. Professor, Ohio University.

HERBERT B. BEBB, Student, University of Oklahoma.

RAYMOND E. BLUME, A.A. (Riverside) Student, University of California, Riverside.

CHARLES E. BOYNTON, IV, B.A. (Emory) Grad. Asst., Emory University.

ALLAN U. BRENDER, Student, McGill University.

ALFRED J. BRUEY, Student, Fenn College.

KENNETH C. BULLOCK, B.S. (Oklahoma) Grad. Asst., Oklahoma State University.

JAMES W. CAFKY, Student, University of Oklahoma.

MRS. MARY L. CANTWELL, M.A. (Columbia) Teacher, St. Petersburg Junior College.

FRANCIS W. CARROLL, M.S. (Purdue) Grad. Fellow, Purdue University.

MISS YI CHANG, M.S. (Illinois) Grad. Asst., University of Illinois.

MRS. VIVIAN S. COHEN, M.A. (California) Instr., Sacramento Junior College.

LLOYD M. COOK, Ed.D. (Univ. of California, Berkeley) Professor, Chico State College.

ALEXANDER S. DAVIS, Ph.D. (North Carolina) Mathematician, National Security Agency, Fort Meade, Maryland.

FRANCES E. DAVIS, M.A. (Michigan) Asst. Professor, University of Hawaii.

LLOYD D. DAVIS, A.B. (Ohio Northern) Grad. Asst., University of Miami.

RAYMOND A. DICKEY, M.A. (Colorado State) Teacher, Central High School, Pueblo, Colorado.

ROBERT D. EAGLETON, Student, Abilene Christian College.

EDWARD M. EDWARDS, Student, University of British Columbia.

SHELDON J. EINHORN, M.A. (Pennsylvania) Asst. Instr., University of Pennsylvania.

PAUL R. ELBERT, Student, Butler University.

ROBERT ELLIS, Ph.D. (Pennsylvania) Asst. Professor, University of Pennsylvania.

FRANCIS J. FELIX, A.B. (Lycoming) Instr., Pennsylvania State University.

MRS. VIVIAN J. FIELDER, M.S. (Illinois) Grad. Asst., University of Illinois.

ROBERT M. FOOTE, M.A. (Texas) Mathematician, American Oil Co., Texas City, Texas.

CLYDE G. FORD, B.A. (Arkansas State) Teaching Asst., University of Oklahoma.

RICHARD K. FOSU, B.S. (McGill) Grad. Student, University of Toronto.

WALTER GAUTSCHI, Dr. (Basle) Mathematician, National Bureau of Standards, Washington, D. C.

FLORENCE E. GILLHAM, M.A. (Columbia) Teacher, St. Petersburg Junior College.

GERALD K. GOFF, M.Ed. (Phillips) Asst. Professor, Southwestern State College.

ROBERT V. GOORDMAN, Electronics Circuit Designer, Bell Telephone Laboratories, South Orange, New Jersey.

PETE A. GRACIA, Student, Lamar State College of Technology.

WAYNE W. GUTZMAN, Ph.D. (Iowa) Professor, University of South Dakota.

JACK L. HARRIS, B.S. (Georgetown) Grad. Asst., University of Kentucky.

EDITH M. HESS, M.L. (Houston) Data Analyst Specialist, Telecomputing Corp., Holloman AFB, New Mexico.

BILLY F. HOBBS, M.A. (Ball S.T.C.) Asst. Professor, Olivet Nazarene College.

GEOFFREY HORROCKS, M.A. (Oxford) Mathematician, Northern Life Assurance Co. of Canada, London, Ontario.

JOHN M. HORVATH, Ph.D. (Budapest) Asst. Professor, University of Maryland.

LAWRENCE C. HOUSE, A.B. (Boston) Asst. Instr., University of Connecticut.

DONALD F. JORDAN, Student, Texas Technological College.

ALVERN W. KAUFMANN, M.A. (Ohio S.U.) Professor, Roberts Wesleyan College.

FERDINAND KERTES, D.Sc. (Rutgers) Hd., Dept. of Math., Perth Amboy High School, New Jersey.

MIDDLETON LAMBERT, B.S. (London) Principal, Texada Elementary-Senior High School, Vananda, B. C.

ANTHONY T. LAURIA, M.S. (Purdue) Research Asst., Purdue University.

CLIFFORD A. LONG, M.S. (Illinois) Grad. Asst., University of Illinois.

RAYMOND A. LUTHER, Engineer, Western Electric, Kearny, New Jersey.

ROGER M. LUTZ, B.S. (Mt. Union) Teaching Fellow, University of Cincinnati.

JOHN E. MCDOWELL, Switchman, Bell Telephone Co., Newark, New Jersey.

MRS. FLORENCE D. MESSINGER, B.A. (Wells) Instr., Texas Christian University.

CAREY G. MUMFORD, Ph.D. (Duke) Professor, North Carolina State College.

WILLIAM C. NEMITZ, M.S. (Ohio S.U.) Grad. Asst., Ohio State University.

PAUL T. NUGENT, A.B. (Franklin) Grad. Asst., Miami University.

MRS. INGRID OWREN, M.S. (Oslo) Instr., University of Alaska.

KYU S. PARK, B.S. (Seoul National) Instr., Dong-A University, Pusan, Korea.

JOSEPH A. PETERS, B.S. (St. Joseph's) Grad. Asst., University of Illinois.

WADE A. PETERSON, B.A. (Washington) Instr., University of Alaska.

GIDEON PEYSER, Ph.D. (New York) Instr., Newark College of Engineering.

ROBERT P. PIKUL, M.S. (Minnesota) Operations Research Analyst, United Aircraft Corp., East Hartford, Conn.

RAMAVARAPU R. RAO, M.S. (Banaral Hindu) Grad. Asst., University of Oklahoma.

JACK REYNOLDS, B.S. (Oklahoma) Instr., University of Oklahoma.

JAMES J. RHYNE, Student, University of Oklahoma.

CAROLYN S. RICE, Student, Texas Christian University.

C. DAVID ROBBINS, M.A. (Arkansas) Instr., University of Oklahoma.

RUTH M. ROBERTS, M.S. (Chicago) Asst. Instr., University of Pennsylvania.

MALCOLM S. ROBERTSON, Ph.D. (Princeton) Professor, Rutgers University.

WILFRED J. ROESLER, A.B. (Loyola) Research Asst., University of Minnesota.

FRANK D. RUSSELL, B.A. (DePauw) Data Analysis Engineer, Telecomputing Corp., Holloman AFB, New Mexico.

ROBERT F. RUTSCHOW, B.S. (V.M.I.) Grad. Student, Pennsylvania State University.

NORMAN SCHAUMBERGER, M.A. (C.C.N.Y.) Instr., Cooper Union.

DONALD C. SCOUTEN, Student, Oklahoma State University.

ANTHONY SEPAN, Student, Temple University.

RONALD M. SHELTON, M.S. (Illinois) Grad. Asst., University of Illinois.

WILLIAM A. SIBLEY, M.S. (Oklahoma) Research Asst., University of Oklahoma.

REBECCA E. SLOVER, Student, Georgetown College.

CLIFFORD W. SLOYER, JR., B.A. (Lehigh) Grad. Asst., Lehigh University.

WAYNE E. SMITH, M.A. (U.C.L.A.) Associate, University of California, Los Angeles.

DONALD N. SPOTH, Data Analyst Engineer, Telecomputing Corp., Holloman AFB, N. M.

KARL STROMBERT, M.A. (Oregon) Acting Instr., University of Washington.

J. ROGER TELLER, B.S. (Cincinnati) Grad. Student, University of Cincinnati.

WILLIAM J. THOMAS, Ph.D. (Michigan State) Asso. Professor, Baylor University.

RALPH N. TOWNSEND, M.S. (Illinois) Asst., University of Illinois.

SHIRLEY R. TREMBLEY, M.A. (Fresno S.C.) Instr., Bakersfield College.

JACK P. TULL, Ph.D. (Illinois) Instr., Ohio State University.

HARRIS W. VAYO, B.A. (Culver-Stockton) Asst., University of Illinois.

GEORGE E. WALLACE, M.A. (Stanford) Teacher, College of San Mateo.

LEROY J. WARREN, Ph.D. (Oregon) Asst. Professor, San Diego State College.

MARTHA F. WATSON, A.B. (Murray S.C.) Grad. Asst., University of Kentucky.

MARTIN T. WECHSLER, Ph.D. (Michigan) Asst. Professor, Wayne State University.

HELEN S. WEIHE, A.B. (Ursuline) Grad. Student, University of Kentucky.

KENNETH J. WHITCOMB, M.A. (Nebraska) Instr., Colorado State University.

BILL WILSON, Student, Baker University.

DAVID G. WILSON, Student, University of Oklahoma.

NEW DEPARTMENT

It is proposed to establish a department of the MONTHLY that would be concerned with mathematics education. This department would include news about current efforts to revise the high school and college curriculum in mathematics, about problems connected with teacher training and certification, and other similar activities. Until an associate editor is appointed to conduct the department, articles may be sent to the editor, Professor R. D. James.

THE NOVEMBER MEETING OF THE PHILADELPHIA SECTION

The annual meeting of the Philadelphia Section, Mathematical Association of America was held at Haverford College on Saturday, November 30, 1957 with Professor Albert Wilansky, Lehigh University, Chairman of the Section, presiding. There were 67 present, including 59 members of the Association.

Dr. I. E. Block, Burroughs Corporation, was elected Chairman of the Section for 1957–58. Professor D. W. Western, Franklin and Marshall College, was elected to the Executive Committee.

The matter of participation by this Section in the National Mathematics Contest for high schools was discussed and the following action taken:

(1) The Philadelphia Section approves in principal that we participate in the Mathematics Contest for high schools in our region.

(2) The chairman shall appoint a committee to work out details with power to act for the Association. This committee shall plan to hold these contests beginning in March of 1959, and shall decide on methods of administration, publication, and awards. It shall, by March 30, 1958, submit its recommendations by mail to the entire membership for comments.

The committee appointed consists of Professors W. S. Lawton (Chairman), T. L. Koehler, E. R. Mullins, Jr., G. C. Webber and D. W. Western.

The following papers were presented at this meeting:

1. *Experimental statistics—some of the concepts and mathematical requirements*, by Dr. Stuart Hunter, American Cyanamid Company and Princeton University, introduced by the Secretary.

2. *Mass distributions on the circle and convex conformal maps*, by Professor I. J. Schoenberg, University of Pennsylvania.

An elementary method is presented which yields the solution in a number of special problems such as the following: Let $1<n_1<n_2<\cdots$ be a given infinite sequence of integers. (1) To find among all positive definite sequences $\{\mu_n\}(n=0,\pm1,\pm2,\cdots)$ with the properties $\mu_0=1$, $\mu_{n_1}=\mu_{n_2}=\cdots=0$, the one which maximizes $|\mu_i|$. (2) For the class of power series $w=F(z)=z+c_{n_1}z^{n_1}+c_{n_2}z^{n_2}+\cdots$ which map $|z|<1$ onto a convex and univalent domain D, to find the largest circle $|w|<\rho$ which is covered by all these convex images D. The paper will appear shortly in the *Proceedings of the Netherlands Academy of Sciences*.

3. *A report on the recommendations of the Commission on Mathematics of the College Entrance Examination Board*, by Professor A. W. Tucker, Princeton University.

4. *Mathematics at a National Science Foundation Summer Institute*, by Professor David Rosen, Swarthmore College.

Professor Rosen's course on "Axiomatics of Number Systems" was given at the University of Pennsylvania in the summer of 1957. It was designed specifically for high school teachers.

G. C. WEBBER, *Secretary*

THE FEBRUARY MEETING OF THE LOUISIANA-MISSISSIPPI SECTION

The thirty-fifth annual meeting of the Louisiana-Mississippi Section of the Mathematical Association of America was held at Loyola University, New Orleans, Louisiana on February 21–22, 1958. The Friday afternoon meeting was held in two concurrent sessions. Professor N. A. Childress, Mississippi Vice-Chairman, and Professor S. M. Spencer, Louisiana Vice-Chairman, presided. Professor A. C. Grimes of Mississippi State College, Chairman of the Section, presided at the Friday evening and Saturday morning sessions. There were 157 persons registered, including 65 members of the Association.

The following officers were elected for the coming year: Chairman, Professor C. G. Killen, Northwestern State College; Vice-Chairman for Mississippi, Professor S. R. Knox, Millsaps College; Vice-Chairman for Louisiana, Professor T. F. Mulcrone, S. J., Loyola University; Secretary-treasurer, Professor T. L. Reynolds, Millsaps College.

At the business meeting Professor H. T. Karnes of Louisiana State University gave a report of the committee on the High School Mathematics Contest, recommending methods for administering the contest within the section. The report was accepted.

The invited speaker for the Friday night meeting was Professor A. H. Clifford of Newcomb College, Tulane University who spoke on "Semigroups." The Saturday morning session heard a panel discussion on "The Undergraduate Mathematics Program at Tulane" by Professor A. D. Wallace, Mathematics Department, Moderator, Dean P. V. Grambsch, Business Administration, Dean L. H. Johnson, Engineering and Dr. C. W. Wing, Director of Admissions.

The following papers were presented:

1. *The High School Mathematics Contest*, by Professor W. H. Fagerstrom, Executive Director, The National Contest, Pan American College.

The speaker gave a brief history of the contest since its beginning in 1949 and discussed its national and international scope and pointed out that 73,000 high school students are scheduled to participate in the contest this year. The plan for giving awards was discussed and it was pointed out that a medal would be given to the leading student in each high school participating.

2. *Old curves, new definitions*, by Professor V. B. Temple, Mississippi Southern College.

A pedal curve is defined as the path traced by the vertex of a right angle as it moves according to specified geometric conditions. The following were defined as pedal curves: (1) The straight line, continuous and closed; (2) The parabola, continuous and closed; (3) The hyperbola as orthic (equilateral), and oblique (nonequilateral); (4) The hypo- and epi-cycloids; (5) The so-called rose curves (roses and clovers); (5) The modified Longchamps curve; (7) The cissoid of Diocles and the strophoid as subpedal curves.

3. *The graduate program in mathematics at Tulane University*, by Professor Fred B. Wright, Tulane University.

The current graduate program at Tulane has been developed since 1947. It is designed to lead a student to the areas of current research as quickly as possible. At the same time, it is so organized as to permit students of varying backgrounds to be inserted into the appropriate place in the curriculum. The success of the program can be measured by comparing the backgrounds of the persons who have been awarded the doctorate to the positions they have held since graduation.

A more recent program is embodied in the degree of Master of Arts in Teaching in Mathematics. This program is designed for teachers in secondary and primary school mathematics, and is administered with the advice of the Department of Education.

4. *A method of expansion of determinants*, by Professor S. R. Knox, Millsaps College.

A compact method of evaluating determinants is exhibited, noting advantages and disadvantages relative to similar methods.

5. *A note on the teaching of infinite series*, by Professor S. B. Murray, Mississippi State College.

In the usual textbook in Calculus the conditions for convergence of an alternating infinite series are given as (1) the Nth term must approach zero as N approaches infinity, and (2) the absolute value of the $N+1$ term must be less than the absolute value of the Nth term, after a certain N. Students often raise a question about the second condition, thinking it must follow from the

first. Two examples are presented to show that the first condition may be satisfied while the second is not satisfied.

6. *An experiment in large-section teaching at Louisiana State University*, by Professor F. A. Rickey, Louisiana State University.

A report on the teaching of Trigonometry in large sections of 180 each with two hour-lectures a week and small group discussions once a week. The results were favorable when compared with those of regular-sized trigonometry classes.

7. *Finding the sums of certain series*, by Professor W. B. Temple, Louisiana Polytechnic Institute.

The fact that $1^3+\cdots+n^3=n(n+1)(2n+1)/6$ can be easily proved by mathematical induction. In this paper this familiar formula is derived. The method used may be applied to deriving formulas for sum of cubes, fourth powers, *etc.*, of the positive integers up to n as well as $\sum_{i=1}^{n} 1/(i(i+1))$ and a number of other finite sums.

8. *Continuous solutions of two functional equations*, by Professor R. D. Boswell, Jr., Mississippi State College.

Let a, A, and k be real numbers where $A>0$. The author shows that the only continuous solutions of the equation $f(x+y)=f(x)+f(y)+a(1-A^x)(1-A^y)$ are those of the form $f(x)=kx-a(1-A^x)$ and the only continuous solutions of the equation $f(x+y)=A^xf(y)+A^yf(x)$ are those of the form $f(x)=kxA^x$.

9. *Mathematics and high-speed computers*, by Professor B. B. Townsend, Louisiana State University.

This paper was an expository presentation of more or less pertinent remarks concerning the mathematics used in digital-computer work, the author's views on what we should teach in this regard, and perhaps some very elementary explanations of what a digital computer does.

10. *The target-missile problem on the electronic-analog computer*, by Professor Margaret M. LaSalle, Southwestern Louisiana Institute.

The object is to plot a trajectory as a missile approaches a moving target. The differential equations of motion are set up, two-dimensional case. A servo-mechanism is used to convert from polar to rectangular coordinates so that an xy-plot is recorded for different initial conditions and angles of attack. Ascertained from this data is the curve requiring the shortest time for a hit.

11. *An analogue of the Chinese remainder theorem in group theory*, by Professor Eugene Schenkman, Louisiana State University, introduced by the Secretary.

For $i=1,\cdots,n$ let C_i be a collection of normal subgroups of G; and let B_i be the intersection of all the $C_j, j=1,\cdots,n, j\neq i$. Then the following two statements are equivalent: (1) If $a_1,\cdots a_n$ are arbitrary elements of G then the intersection of the cosets a_iC_i for $i=1,\cdots,n$ is not empty, and (2) $G=B_1B_2\cdots,B_n$.

T. L. REYNOLDS, *Secretary*

THE MARCH MEETING OF THE SOUTHEASTERN SECTION

The annual meeting of the Southeastern Section of the Mathematical Association of America was held March 14-15, 1958, at the University of Florida, Gainesville, Florida. Professors Trevor Evans, Chairman of the Section; D. E. South, Vice-Chairman; F. W. Kokomoor, Sectional Governor; and E. H. Hadlock presided over the general and divisional sessions. There were 219 in attendance, including 139 members of the Association.

The following officers were elected for the coming year: Chairman, Professor D. E.

South, University of Florida; Vice-Chairman, Professor T. C. Carson, East Tennessee State College; Secretary-Treasurer, Professor H. A. Robinson, Agnes Scott College. The Section accepted the High School Contests Committee's report that at this time; since several states have a contest of their own, the Section should not attempt to administer the national contest. It was felt that both the decision to participate in and the administration of the contest should be on the state level. A new committee was named, each member being a state director except the chairman: Professor J. H. Banks (chairman), Professor Mariano Garcia, Jr. (Puerto Rico), Miss E. M. Lynch (Georgia), Professor J. D. Mancill (Alabama), Professor E. D. Nichols (Florida), Professor J. W. Sawyer (North Carolina), Professor C. E. Shuler (South Carolina) and Professor F. L. Wren (Tennessee).

As was done some twenty years ago, the Secretary arranged a display of 63 mathematics books published by members of the Southeastern Section.

The following program was presented:

1. *An algebraic solution of an extreme value problem in Euclidean n-space*, by Professor G. B. Huff, University of Georgia.

A fundamental problem in factor analysis may be reduced to the following problem in Euclidean n-space. If $z_1, \cdots, z_n$ is a set of unit vectors and $r<n$, what is the space S_r of dimension r which most nearly contains $z_1, \cdots, z_n$? It is shown by algebraic methods that the S_r for which the sum of the squares of the distances to $z_1, \cdots, z_n$ is a minimum is spanned by the r largest characteristic vectors of a matrix R, where r_{ij} is the cosine of the angle between z_i and z_j.

2. *A construction for some finite geometries*, by Professor J. R. Wesson, Vanderbilt University.

The finite plane with $n+1$ points on each line has points which may be named $1, 2, 3, \cdots, n^2+n+1$. In case n is a prime, the points on each line may be listed in a direct and straightforward manner. This construction is accomplished without "trial and error" and without using coordinates other than the labels $1, 2, \cdots, n^2+n+1$.

3. *A new proof of the theorem of mean value*, by Mr. Chung-Lie Wang, University of South Carolina.

A proof of the mean value theorem is given based on translation and rotation of axes, without any appeal to geometric intuition.

4. *Keeping track of Explorer*, by Dr. C. L. Bradshaw, Computation Laboratory, ABMA, Huntsville, Alabama.

The problem of tracking an artificial earth satellite demands that one possess computational procedures which are extremely fast and highly accurate. One must first determine (from launching conditions, first observations, *etc.*) a good set of initial conditions, and then provide flexible means of continually correcting the orbit as further observations are made. The ABMA approach used various methods of numerical integration to define the ephemeris and methods of partial derivatives and least squares for correction.

5. *On the commutative and associative laws*, by Professor Trevor Evans, Emory University.

This paper is a study of the types of identities which can be satisfied by algebraic systems with a binary operation. It is shown that no nontrivial group exists satisfying identities incompatible with commutativity or with finiteness. However, loops exist satisfying identities incompatible with associativity and with finiteness. Another question treated is the existence of an infinite irredundant set of identities compatible with associativity.

6. *Some recent developments in the undergraduate mathematics curriculum and their implications for the future*, by Professor H. E. Taylor, Florida State University.

Widespread calls for changes seem to be based on: (1) the fact that the profound mathematical advances of this century have had little effect on the curriculum; (2) the use by biological and social sciences of an increasing amount of new mathematics; (3) increasing concern for mathematics as part of liberal education. Developments which do more than rearrange traditional material are meeting varying degrees of success. Introduction of the ideas of sets and relations early makes it possible to introduce some "modern" developments and bring clarity, precision, insight, and efficiency to the study of solutions of equations, inequalities, analytic geometry and the function concept.

7. *An important responsibility of every university mathematics department*, by Professor W. A. Gager, University of Florida.

In most universities the education college depends upon the mathematics department of the liberal arts college to provide adequate mathematical training for those who plan to teach in high school. This paper raises questions concerning the mathematical content of the courses offered and the effectiveness with which the materials are presented to the prospective teachers. Modernization of the curricular offerings and extra-special attention to those who plan to be our future teachers of mathematics are urged.

8. *Innovations in mathematics teaching at North Carolina State College*, by Professor C. G. Mumford, North Carolina State College.

Activities recently initiated include the following: (a) a superior student program beginning at the freshman level, with some acceleration, but with emphasis on depth; (b) an honors program for engineering seniors conducted jointly by an engineering department and the mathematics department; (c) course work and research in computer mathematics with availability of analog and digital computers; (d) use of lecture-section teaching with about 90 students per section in calculus and introductory differential equations.

9. *Whither secondary mathematics?* by Professor E. D. Nichols, Florida State University.

The author discusses implications of the recommendations of the Commission on Mathematics for the future secondary-mathematics curriculum. The program in Algebra now in operation with selected eighth-graders at the University School of the Florida State University is considered as an example of an elementary-algebra course in which emphasis is placed on the structure of algebra rather than on manipulative skills.

10. *The stress distribution in a rotating limaçon*, by Professor C. B. Smith, University of Florida.

A thin plate bounded by the curve $r=b-2a\sin\theta$ $(b>2a)$ is assumed to be rotating about the y-axis at a constant angular velocity. The stresses arising in the plate are due to a centrifugal force or body force. The body force is removed from the problem by obtaining a particular solution, and the solution of the reduced problem is then found by using complex variables. The superposition of the two solutions gives the complete solution for the problem. When the constant a is set equal to zero, the solution readily reduces to the case of a rotating circular plate.

11. *A note on the synchronization problem*, by Professor Stephen Kulik, University of South Carolina.

Machines which are in operation are supervised. The number of operators is less than the number of machines. If a machine stops it is attended by the first available operator. The ratio of the time during which a machine is running productively to the total period of operation is its effi-

ciency. The solution for efficiency is given in the form of an algebraic equation of degree equal to the number of machines. It is assumed that (a) number of stops is proportional to productive running time; (b) the total time for repairs is small; (c) the distribution of the number of machines running productively at any moment follows approximately the binomial law.

12. *Optimal filtering and prediction*, by Professor Andrew Sobczyk, University of Florida.

A signal $\{z_k\}$ (discrete time series) consists of a desired message $\{x_k\}$ plus a noise $\{c_k\}$. It is required to determine a system of weights (filter) $\{w_n\}$ such that if $\{z_k\}$ is applied as input, the output $y_k=\sum_{n=0}^{N} w_n z_{k-n}$ will represent as closely as possible future values $\{x_{k+h}\}$ of the message. For the minimum mean-square difference criterion of optimality, the problem has a solution in terms of correlations which depend on infinite past and future of the time series. This paper contributes to the similar problem where, as in any practical case, only a finite amount of data is available.

13. *Some numerical results concerning the asymptotic distribution of Spearman's rank correlation coefficient*, by Professor N. C. Perry, Alabama Polytechnic Institute.

The average error per class interval (E) of an asymptotic probability function of Spearman's ρ coefficient was computed by comparison with the exact distribution for small sample sizes. The use of log log paper indicated a relation between E and sample size n approximately of the form $E=k/n^a$. Least square estimates of k and a show that to obtain probabilities of three decimal accuracy samples of size $n\geqq 11$ should be used; for five decimal accuracy $n\geqq 23$.

14. *Problem of Pellian equations*, by Professor J. W. Greiner, University of Florida.

Theorems and corollaries were given describing all integral solutions of the equation $\delta x^2-dy^2=1$, with $\delta\cdot d=D$, in terms of $t_n+u_n\sqrt{D}$, solutions of Pell's equation, $t^2-Du^2=1$, when t_1 is odd and D is a product of distinct odd primes.

15. *A note on Ky Fan's extension of the Bernstein theorem*, by Mr. B. L. Sanders, Florida State University.

F. P. Callahan and S. G. Kneale have successfully characterized the decompositions which will serve to prove the Bernstein theorem in the classical manner (*A note on the Schroeder-Bernstein theorem*, this MONTHLY, vol. 64, 1957, pp. 423–424). The Bernstein theorem was generalized by S. Banach and this theorem was then extended by Ky Fan (*Note on a theorem of Banach*, Mathematische Zeitschrift, vol. 55, 1952, pp. 308–309). The present paper discusses Ky Fan's extension in the light of the first paper mentioned above.

16. *Nonasymptotic semiorbits under expansive homeomorphisms*, by Professor B. F Bryant, Vanderbilt University.

Let X be a compact, self-dense metric space, and let f be an expansive self-homeomorphism of X (for terminology, see Gottschalk and Hedlund, *Topological Dynamics*, A.M.S. Coll. Pub., Vol. 36). It is shown that for each $x\in X$ and each $\epsilon>0$, there exists $y\in U(x,\epsilon)$ such that $0(x)$ and $0(y)$ are not doubly asymptotic. An example is given to show that the conclusion does not necessarily hold if X is not selfdense.

17. *On the Diophantine homogeneous quadratic equation* $f(x_1, x_2, x_3)=a$, by Professor E. H. Hadlock, University of Florida.

It is shown that the equation whose left member is a ternary quadratic form f with mild restrictions on its coefficients, and whose right member is a given integer, has solutions if any one of the three equations E, obtained from f by the elimination of its cross-products has a solution, provided the variables in E are subject to proper choice of signs and conversely.

18. *Some uses of linear spaces in analysis*, by Professor F. A. Ficken, University of Tennessee.

The speaker began this expository talk by an examination of the Picard theorem for the equation $y' = f(x, y)$, showing that the proof amounted to finding, by an iterative process, a fixed point of a mapping into itself of a suitable complete metric linear space. After some remarks on the algebraic and topological characteristics of infinite-dimensional linear spaces and their duals, he confined attention to Banach spaces. Illustrations included a boundary value problem, an integral operator, and the Euler expression for a variational problem. Many useful properties of a Hilbert space H were seen to be connected with the fact that H is self-dual. Examples included Fourier's series and integral and the spectral theorem for a symmetric (self-adjoint) transformation.

19. *A note on a development of logarithms using the function concept*, by Professor C. L. Seebeck, Jr. and Mr. H. C. Miller, Jr., University of Alabama, presented by Professor Seebeck.

The recent paper *A development of logarithms using the function concept*, this MONTHLY, vol. 64, 1957, pp. 667–668, is extended to include elementary proofs that the function log as defined is a unique, reversible function from R^+ onto R.

20. *A note on an involution of period thirteen*, by Professor W. R. Hutcherson, University of Florida.

A certain quintic surface invariant under a cyclic homography of period thirteen is investigated at the point (1, 0, 0, 0). Certain invariant curves on this surface and paseing through this point were studied, both by the classical method and by the method of Godeaux.

21. *Circulants and their groups*, by Professor F. A. Lewis, University of Alabama.

The purpose of this paper is to determine the order of the largest permutation group leaving certain circulants invariant.

22. *The Fibonacci numbers and a dissection of a square*, by Cadet W. R. Alford, The Citadel.

A square of side u_n (the nth term of the Fibonnacci sequence) is divided into two congruent right triangles and two congruent trapezoids which can be reassembled in a rectangle form of dimensions u_{n-1} by u_{n+1} whose area differs from that of the square by unity.

23. *Some observations on the method of conjugate gradients*, by Dr. A. S. Householder, Oak Ridge National Laboratory.

A complete matricial formulation of the method is presented, which exhibits it in form somewhat more perspicuous than the usual scalar representation, and its relation to other biorthogonalization techniques is pointed out. The primary objective, however, is to exhibit effective bounds on the quantities arising in the computation, obtained in terms of a "condition number" of the matrix to be inverted. In cases where this condition number can be obtained, these bounds provide the scale factors needed for carrying out the computation in fixed point.

24. *Mathematics and biology*, by Mr. G. R. Flowers, Emory University.

This paper is a review of N. Rashevsky's *Topology and life: in search of general mathematical principles in biology and sociology*, Bulletin of Mathematical Biophysics, vol. 16, pp. 317–348, 1954. Emphasis is given to the idea of evolution as transformations of a directed graph.

25. *Moore-Smith limits in elementary calculus*, by Professor Herman Meyer, University of Miami.

The author describes a course in introductory calculus taught at the University of Miami.

Moore-Smith limits are introduced and basic limit theorems are proved in this general environment. They are then specialized to obtain the derivative and the integral.

26. *Taylor's series for complex variables*, by Mr. Chung-Lie Wang, University of South Carolina.

In the usual derivation of Taylor's series for a function of a complex variable Cauchy's integral formula is ordinarily employed. In this paper the theorem is established by using Taylor's theorem for a function of two real variables and without the use of Cauchy's integral formula.

27. *Solution of a class of quasilinear partial differential equations of first order*, by Professor Diran Sarafyan, University of Florida.

Let $A_i=a_ix^r+b_iy^r+c_iz^r+d_i$ where a, b, c, d and r are real constants, and $C(abc)$ and $D(abd)$ are third-order determinants made of indicated elements. Consider the quasilinear partial differential equations of first order $A_1p+A_2q=A_3$. The author shows that if C and D are zero then a class of particular solutions is given by $F_1=C_1$ where F_1 is the determinant obtained from $C(abc)$ when c_1, c_2, and c_3 are replaced by x^r, y^r and z^r respectively and C_1 is an arbitrary constant. The consideration of $F_1=C_1$ together with the system $dx/A_1=dy/A_2=dz/A_3$ makes possible the determination of another class of independent particular solutions $F_2=C_2$. The class of all solutions are represented by $F_1=g(F_2)$ where g is an arbitrary function.

28. *Criteria for polynomial solutions of a class of linear self-adjoint differential equations*, by Professor R. W. Cowan, University of Florida.

The linear self-adjoint differential equation is taken in such a form that the recurrence relation obtained by Frobenius' method will contain only two terms. All the singular points of the differential equation are regular. In order that one series solution reduces to a polynomial, conditions are imposed on the coefficients of the differential equation that at least one root of the indicial equation be a nonnegative integer and that the resulting series solution terminate.

29. *Differential equations whose coefficients involve q-periodic functions*, by Professor Tomlinson Fort, University of South Carolina.

Professor Fort discussed briefly q-periodic functions and almost-periodic functions. He then discussed the nature of the solution of certain differential equations whose coefficients involve q-periodic functions. He pointed out some of the problems involved in the study of differential equations with almost-periodic coefficients.

30. *Another proof of the fundamental theorem of algebra*, by Professor M. K. Fort, Jr., University of Georgia.

The notion of continuous square root of a mapping into the complex plane was introduced. Several basic theorems concerning this concept was stated, with outlines of proofs. A new proof of the fundamental theorem of algebra was then obtained by making use of these basic theorems about continuous square roots of mappings.

31. *On the characteristic roots of certain related matrices*, by Professor J. W. Ellis, Florida State University.

If $z=x+iy$ is any complex number, let $f(z)$ denote the real 2×2 matrix whose rows are $(x, -y)$ and (y, x). Now if M is any $n\times n$ matrix of complex numbers, let M' denote the $2n\times2n$ real matrix obtained by replacing in M each m_{ij} by the elements of the matrix $f(m_{ij})$. Then the characteristic roots of M' are precisely those of M, plus their conjugates. A proof of this result is given using vector-matrix multiplication only. A by-product is a simple expression for characteristic vectors of M' in terms of the corresponding characteristic vectors of M.

32. *A Diophantine system*, by Mr. D. E. Thoro, University of Florida.

An elementary proof is given of the following theorem. A necessary and sufficient condition for the existence of integral solutions of two Diophantine equations in three unknowns is that the

g.c.d. of the second order determinants of the matrix of the coefficients is equal to the g.c.d. for the augmented matrix. Formulas for the general solution are obtained.

33. *A rational-valued metric on the space of rational pairs*, by Professor E. B. Shanks, Vanderbilt University.

A metric is defined over the space of rational pairs after a preliminary definition of a "pseudo square root." The pseudo square root is monotonic, strictly increasing and it is continuous except at the origin. A proof is given that the metric assumes only rational values on the space of rational pairs and assigns distances between pairs of points approximately the same as the Euclidean metric and that this approximation can be made as close as desired by choosing a parameter n involved in the definition large enough. The topology induced by the metric is discrete.

34. *Partly-ordered ideal-preserving groups*, by Mr. J. F. Andrus, University of Florida.

The postulate giving the relationship between the order relation and the group operation of a partly-ordered group (assumed to be additive) is replaced by a postulate which states that the sum of any element and any order ideal of the system must be either an order ideal or an order dual ideal. The principal theorem relates many properties of the resulting system to those of a certain normal subgroup which forms a partly-ordered group.

35. *A note on the conics*, by Professor Jose Gallego-Diaz, Vanderbilt University.

The function of a complex variable $w=z^2$ is used to map a system of conics which have the origin for focus into another system of conics which have the origin for center. More than twenty-five original examples are given of the use of this transformation in problems involving locus, envelopes, graphical constructions, maximum and minimum, orthogonal trajectories and so forth.

H. A. ROBINSON, *Secretary*

THE MARCH MEETING OF THE SOUTHERN CALIFORNIA SECTION

The thirty-eighth regular meeting of the Southern California Section of the Mathematical Association of America was held at Pasadena City College, Pasadena, California, on March 8, 1958. Professor P. B. Johnson, Chairman of the Section, presided. The registered attendance was 142, including 105 members of the Association.

At the business meeting the following officers were elected for the next academic year: Chairman, Professor P. J. Kelly, Santa Barbara College, University of California; Vice-Chairman, Professor D. H. Hyers, University of Southern California; Secretary-Treasurer, Mr. R. B. Herrera, Los Angeles City College. The following members were elected to the Program Committee for the coming year: Professor F. A. Valentine (Chairman), University of California, Los Angeles; Professor Lulu Bechtolsheim, University of Redlands; Professor S. E. Urner, Los Angeles State College; and Professor P. B. Johnson, Occidental College.

The following program was presented:

1. *Electromagnetic theory without Maxwell's Equations*, by Professor R. M. Redheffer, University of California, Los Angeles.

Interaction of transducers in a transmission line induces an algebra through which a "linear network" can be defined. Applied to a homogeneous dielectric sheet this algebra gives nonlinear functional equations for the transmission and reflection. The solution agrees with the classical results, when two complex constants that arise are related suitably to ϵ, μ, λ, θ. For an inhomogeneous sheet the process gives a Riccati equation for the right-hand reflection, and shows how the left-hand reflection and transmission follow therefrom by quadrature. This physical model suggests interesting properties of the general Riccati equation, which can be verified independently.

2. *Unimodular matrices with integral elements*, by Dr. Olga Taussky, California Institute of Technology, introduced by the Secretary.

This paper is a discussion of the following theorem and its generalizations: Every Abelian subgroup of the modular group is cyclic. The results are due to numerous authors.

This theorem can be interpreted not only as a result in number theory and matrix theory, but also as a result in abstract group theory or in non-Euclidean geometry. Analogous theorems on groups of matrices with elements in a fixed algebraic number field have also been investigated.

3. *Implications for college mathematics departments of the offering of sections in mathematical analysis in the senior high schools*, discussed by a panel of five persons.

The members of the panel discussing the announced topic were: Professor May M. Beenken, Immaculate Heart College, Chairman; Dr. Marian Cliffe, Los Angeles City Schools; Mr. Gerald Baughman, Pacific High School, San Bernardino; Professor C. G. Jaeger, Pomona College; Professor Clifford Bell, University of California, Los Angeles.

Senior courses in mathematical analysis in the Los Angeles city high schools are not intended to replace freshman college courses, but to prepare students better for these courses by teaching unifying concepts of mathematics. Such courses provide a simple introduction to concepts to be developed more fully in college; they help to develop skills in note-taking and study-reading of mathematical materials. Typical of a more rigorous course in elementary college mathematics is that successfully being offered to qualified seniors at Pacific High School, San Bernardino. Colleges welcome the introduction of such courses in the high school, and they will allow freshmen, who pass a placement test, to enroll in more advanced college mathematics courses.

4. *Measure theory by means of metric space methods*, by Professor H. G. Tucker, University of California, Riverside.

The development of measure and the Lebesgue integral is presented from a unified, metric space point of view. Both the unique extension of a sigma-finite measure on an additive class of sets to the completely additive class generated by this additive class and the definition of the Lebesgue integral are obtained by means of completion spaces of appropriate metric spaces. Various methods of integration (especially of functions taking values in a Banach space) can be compared by ordering according to the strengths of the corresponding metric topologies determining these integrals.

5. *Some mathematical problems in determining satellite orbits*, by Dr. S. E. Benesch, Jet Propulsion Laboratory, California Institute of Technology, introduced by the Secretary.

Work done under the author's direction was used to determine the orbit of the Explorer 1 satellite. Techniques used in preliminary orbit description (lower bound on perigee) before completion of the first pass were outlined. Global properties of the Doppler curve were developed to relate the change in altitude from horizon to horizon with asymetrical asymptotes, under an approximation that the product of velocity and distance from the earth's center is constant. Local properties of the Doppler curve relate the time of closest approach, $f(0)$, and the time for the point of inflection. Using Taylor's expansion in $f(0)$ for a slant range of 1000 miles and an acceleration of $\frac{1}{4}$ mile/sec.^{2}, it was found that the time difference was 5.5 sec., which was an important correction in refining the ephemeris by the use of Doppler data.

6. *Infinite-valued propositional logic*, by Professor C. C. Chang, University of Southern California, introduced by the Secretary.

In this paper algebraic systems called MV-algebras are introduced and studied. The study of such algebras is useful in establishing an algebraic proof of the completeness of the infinite-valued propositional logic of Łukasiewicz. Previously the completeness was proved only by methods from logic and metamathematics. Briefly, every MV-algebra is a subdirect product of linearly

ordered MV-algebras, and every linearly ordered MV-algebra is isomorphically embeddable into a segment of an ordered Abelian group. Since any two ordered Abelian groups are universally equivalent, every identity that holds in the segment $[0, 1]$ of the reals must also hold for every MV-algebra. This paves the way to the algebraic proof of completeness.

7. *A new college of science*, by Professor R. C. James, Harvey Mudd College, Claremont.

Harvey Mudd College, the fifth of the Associated Colleges at Claremont, California, admitted its first freshman class in 1957. It was founded in the belief that there is a need for scientists and engineers with broad training in the humanities and social studies. Majors are offered in mathematics, physics, chemistry, and engineering science. The engineering science major will emphasize basic engineering principles, rather than specialized applications. The mathematics program has been planned to provide a calculus course carefully coordinated with the physics and chemistry courses. Upper division courses serve nonmajors as well as majors, giving a broad introduction to modern mathematics for majors, including topology, modern algebra, and operations analysis.

R. B. HERRERA, *Secretary*

CALENDAR OF FUTURE MEETINGS

Thirty-ninth Summer Meeting, Massachusetts Institute of Technology, Cambridge, Massachusetts, August 25–28, 1958.

Forty-second Annual Meeting, University of Pennsylvania, Philadelphia, Pennsylvania, January 22–23, 1959.

The following is a list of the Sections of the Association with dates of future meetings so far as they have been reported to the Associate Secretary.

ALLEGHENY MOUNTAIN

ILLINOIS

INDIANA

IOWA

KANSAS

KENTUCKY

LOUISIANA-MISSISSIPPI, Buena Vista Hotel, Biloxi, Mississippi, February 13–14, 1959.

MARYLAND-DISTRICT OF COLUMBIA-VIRGINIA, George Washington University, Washington, D. C., December 6, 1958.

METROPOLITAN NEW YORK

MICHIGAN

MINNESOTA

MISSOURI

NEBRASKA

NEW JERSEY, Rutgers University, New Brunswick, November 1, 1958

NORTHEASTERN, College of the Holy Cross, Worcester, Massachusetts, November 29, 1958.

NORTHERN CALIFORNIA, University of California, Berkeley, June 17, 1958 (joint meeting with ASEE, mathematics division).

OHIO

OKLAHOMA, Oklahoma City University, October 24, 1958.

PACIFIC NORTHWEST, Oregon State College, Corvallis, June 20, 1958.

PHILADELPHIA, Lehigh University, Bethlehem, November 29, 1958.

ROCKY MOUNTAIN

SOUTHEASTERN, East Tennessee State College, Johnson City, March 20–21, 1959.

SOUTHERN CALIFORNIA, University of Redlands, Redlands, March 14, 1959.

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IBM CAREER OPPORTUNITIES

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Qualifications: M.S. or Ph.D. in mathematics or physics and three to five years' experience.

COMPUTER ANALYST to make math analyses of inertial control systems. Should be able to understand and evaluate various alternate computational approaches to the solution of control problems. Will create mathematical models of bombing and navigational systems for solution by a real-time digital computer and will analyze and evaluate such models on the basis of system performance and computational accuracy.

Qualifications: Ph.D. or M.S. in Physics or Engineering Science with strong math background and one to four years' experience in using digital techniques to solve real-time control problems.

SENIOR PROGRAMMER to analyze engineering problems and develop machine programs to solve them by digital computers. Should be able to produce excellent machine programs and grasp the techniques of creating programs for digital computer. Will develop digital programs to solve bombing and navigation systems in real time and to evaluate such programs for control systems by means of simulation on the 704 data processing machine.

Qualifications: M.S. in Physics or Engineering Science with strong math background and at least two years' experience solving problems by digital computer, preferably in the field of real-time control.

DIAGNOSTIC PROGRAMMER to write programs to do diagnostic work for a real-time digital computer. Should understand what takes place in control systems and be able to write programs to diagnose troubles. Programs are for a real-time digital computer used in bombing-navigational systems and involve use of a 704 DPM to simulate logic of the computer.

Qualifications: M.S. in Physics or Engineering Science with math minor and at least two years' experience in diagnostics or design of digital computers.

CONTROL SYSTEM ANALYST to perform physical and mathematical analyses necessary to solve complex inertial control systems by use of real-time digital computers. Applications in the area of navigational-bombing systems, missile systems, special-purpose computer systems such as DDA, etc.

Qualifications: Ph.D. or M.S. in Physics with strong math minor and up to two years' experience.

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